Neutrinos from GRBs, and test of the cosmic ray paradigm

Philipp Baerwald

Institut für Theoretische Physik und Astrophysik
Universität Würzburg

in collaboration with Mauricio Bustamante, Svenja Hümmer, and Walter Winter

Neutrinos at the Forefront in Lyon
October 23, 2012
GRB Fireballs (What we assume)

- Several seconds long bursts of $\gamma$-rays from one point source
- Most luminous events in the known universe
- Connected to ultra-relativistic expanding fireball, candidates for UHE cosmic ray sources
- Interactions of protons with photons can lead to the production of neutrinos

from www.swift.ac.uk
Photohadronic production of neutrinos

Basic approach by Waxman and Bahcall: approximation of $p\gamma$ interaction cross section using $\Delta$-resonance

$$p + \gamma \rightarrow \Delta^+ \rightarrow \begin{cases} n + \pi^+ & \frac{1}{3} \text{ of all cases} \\ p + \pi^0 & \frac{2}{3} \text{ of all cases} \end{cases}$$

The $\pi^+$ decay producing $\nu_\mu$, $\bar{\nu}_\mu$ and $\nu_e$ in the process

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

Standard conclusion

$\nu$ result from interaction of $p$ and $\gamma$ in GRBs, with a ratio $(\nu_e : \nu_\mu : \nu_\tau)$ of $(1 : 2 : 0)$, or $(1 : 1 : 1)$ after flavor mixing.

See e.g. [Waxman and Bahcall, Phys. Rev. Lett. 78 (12), 2292 (1997)]
The normalization (IceCube approach)

\[ x_{\pi \rightarrow \nu} = \frac{1}{2} \cdot \frac{1}{4} \]

\[
\int_{0}^{\infty} dE_{\nu} E_{\nu} F_{\nu}(E_{\nu}) = \frac{1}{8} \left( 1 - (1 - \langle x_{p \rightarrow \pi} \rangle) \frac{\Delta R}{\lambda_{p\gamma}} \right) \frac{1}{f_{e}} \int_{10 \text{ MeV}}^{1 \text{ keV}} dE_{\gamma} E_{\gamma} F_{\gamma}(E_{\gamma})
\]

Fraction of energy \( f_{\pi} \)

with number of interactions

\[
\frac{\Delta R}{\lambda_{p\gamma}} = \left( \frac{L_{\gamma}^{\text{iso}}}{10^{52} \text{ erg s}^{-1}} \right) \left( \frac{0.01 \text{ s}}{t_{\text{var}}} \right) \left( \frac{10^{2.5}}{\Gamma_{\text{jet}}} \right) ^{4} \left( \frac{\text{MeV}}{\epsilon_{\gamma}} \right)
\]

and average energy lost to pions (per interaction)

\[ \langle x_{p \rightarrow \pi} \rangle \simeq 0.2 \]

from [Abbasi et al., Astrophys. J. 710, 346 (2010)],

based on [Guetta et al., Astropart. Phys. 20, 429 (2004)]
The spectral shape (IceCube approach)

\[ F_\gamma \propto \begin{cases} 
  (\frac{E_\gamma}{\epsilon_{\gamma,b}})^{-\alpha_{\gamma}} & \text{for } E_\gamma \leq \epsilon_{\gamma,b} \\
  (\frac{E_\gamma}{\epsilon_{\gamma,b}})^{-\beta_{\gamma}} & \text{for } E_\gamma > \epsilon_{\gamma,b}
\end{cases} \]

from [Waxman and Bahcall, Phys. Rev. D59, 023002 (1998)]
GRB fireballs in trouble?

LETTER

An absence of neutrinos associated with cosmic-ray acceleration in γ-ray bursts

IceCube Collaboration*

Very energetic astrophysical events are required to accelerate cosmic rays to above 10^{18} electronvolts. GRBs (γ-ray bursts) have been proposed as possible candidate sources. In the GRB ‘fireball’ model, cosmic-ray acceleration should be accompanied by neutrinos produced in the decay of charged pions created in interactions between the high-energy cosmic-ray protons and γ-rays. Previous searches for such neutrinos found none, but the constraints were weak because the sensitivity was at best approximately equal to the predicted flux. Here we report an upper limit on the flux of energetic neutrinos associated with GRBs that is at least a factor of 3.7 below the predictions. This implies either that GRBs are not the only sources of cosmic rays with energies exceeding 10^{18} electronvolts or that the efficiency of neutrino production is much lower than has been predicted.

Neutrinos from GRBs are produced in the decay of charged pions produced in interactions between high-energy protons and the intense γ-ray background within the GRB fireball, for example in the A-resonance process $p + \gamma \rightarrow A^* \rightarrow n + \pi^+$. In the GRB ‘fireball’ model, cosmic-ray acceleration should be accompanied by neutrinos produced in the decay of charged pions created in interactions between the high-energy cosmic-ray protons and γ-rays. Previous searches for such neutrinos found none, but the constraints were weak because the sensitivity was at best approximately equal to the predicted flux. Here we report an upper limit on the flux of energetic neutrinos associated with GRBs that is at least a factor of 3.7 below the predictions. This implies either that GRBs are not the only sources of cosmic rays with energies exceeding 10^{18} electronvolts or that the efficiency of neutrino production is much lower than has been predicted.

As in our previous study, we conducted two analyses of the IceCube data. In a model-dependent analysis, we examine data during the period of γ-ray emission reported by any satellite for neutrinos with the energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.

In the 59-string portion of the model-dependent analysis, no events were found to be both on-source and on time (within 10^{-1} of a GRB and between $T_{\text{start}}$ and $T_{\text{stop}}$). From the individual burst spectra with an energy spectrum predicted from the γ-ray spectra of individual GRBs. The model-independent analysis searches more generically for neutrinos on wider timescales, up to the limit of sensitivity to small numbers of events at ±1 day, or with different spectra. Both analyses follow the methods used in our previous work, with the exception of slightly changed event selection and the addition of the Southern Hemisphere analysis. Owing to the large background of downgoing muons from the southern sky, the Southern Hemisphere analysis is sensitive mainly to higher-energy events (Supplementary Fig. 3). Systematic uncertainties from detector effects have been included in the reported limits from both analyses, and were estimated by varying the simulated detector response and recomputing the limit, with the dominant factor being the efficiency of the detector’s optical sensors.
Contributions to the full photohadronic cross section

Contributions to \((\nu_\mu + \bar{\nu}_\mu)\) flux from \(\pi^\pm\) decay divided in:

- \(\Delta(1232)\)-resonance
- Higher resonances
- \(t\)-channel (direct production)
- High energy processes (multiple \(\pi\))

from [PB, Hümmer, and Winter, Phys. Rev. D83, 067303 (2011)]

Especially ”Multi \(\pi\)” contribution leads to change of flux shape; neutrino flux higher by up to a factor of 3 compared to WB treatment
Spectral features

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu , \]
\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu , \]

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu , \]
\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu , \]

\[ K^+ \rightarrow \mu^+ + \nu_\mu , \]
\[ n \rightarrow p + e^- + \bar{\nu}_e . \]

Spectral features

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu , \]
\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu , \]
\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu , \]
\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu , \]
\[ K^+ \rightarrow \mu^+ + \nu_\mu , \]
\[ n \rightarrow p + e^- + \bar{\nu}_e . \]


Resulting \( \nu_\mu \) flux (at the observer)

\[
\begin{align*}
E^2 \phi_{E, \nu_\mu} &= 
\begin{cases}
10^7 & \text{NeuCosmA 2010} \\
10^8 & \text{Total flux} \\
10^9 & \text{WB flux} \\
10^{10} & \text{from } \pi \\
10^{11} & \text{from } K \\
10^{12} & \text{from } m \\
10^{13} & \text{from } p \\
10^{14} & \text{from } K^+ \\
10^{15} & \text{from } n \\
10^{16} & \text{from } n^+ \\
10^{17} & \text{from } \pi^+ \\
10^{18} & \text{from } \mu^+ \\
10^{19} & \text{from } \pi^- \\
10^{20} & \text{from } \mu^- \\
10^{21} & \text{from } K^- \\
10^{22} & \text{from } n^- \\
10^{23} & \text{from } \pi^- \\
10^{24} & \text{from } \mu^- \\
10^{25} & \text{from } K^- \\
10^{26} & \text{from } n^- \\
10^{27} & \text{from } \pi^- \\
10^{28} & \text{from } \mu^- \\
10^{29} & \text{from } K^- \\
10^{30} & \text{from } n^- \\
10^{31} & \text{from } \pi^- \\
10^{32} & \text{from } \mu^- \\
10^{33} & \text{from } K^- \\
10^{34} & \text{from } n^- \\
10^{35} & \text{from } \pi^- \\
10^{36} & \text{from } \mu^- \\
10^{37} & \text{from } K^- \\
10^{38} & \text{from } n^- \\
10^{39} & \text{from } \pi^- \\
10^{40} & \text{from } \mu^- \\
10^{41} & \text{from } K^- \\
10^{42} & \text{from } n^- \\
10^{43} & \text{from } \pi^- \\
10^{44} & \text{from } \mu^- \\
10^{45} & \text{from } K^- \\
10^{46} & \text{from } n^- \\
10^{47} & \text{from } \pi^- \\
10^{48} & \text{from } \mu^- \\
10^{49} & \text{from } K^- \\
10^{50} & \text{from } n^- \\
10^{51} & \text{from } \pi^- \\
10^{52} & \text{from } \mu^- \\
10^{53} & \text{from } K^- \\
10^{54} & \text{from } n^- \\
10^{55} & \text{from } \pi^- \\
10^{56} & \text{from } \mu^- \\
10^{57} & \text{from } K^- \\
10^{58} & \text{from } n^- \\
10^{59} & \text{from } \pi^- \\
10^{60} & \text{from } \mu^- \\
10^{61} & \text{from } K^- \\
10^{62} & \text{from } n^- \\
10^{63} & \text{from } \pi^- \\
10^{64} & \text{from } \mu^- \\
10^{65} & \text{from } K^- \\
10^{66} & \text{from } n^- \\
10^{67} & \text{from } \pi^- \\
10^{68} & \text{from } \mu^- \\
10^{69} & \text{from } K^- \\
10^{70} & \text{from } n^- \\
10^{71} & \text{from } \pi^- \\
10^{72} & \text{from } \mu^- \\
10^{73} & \text{from } K^- \\
10^{74} & \text{from } n^- \\
10^{75} & \text{from } \pi^- \\
10^{76} & \text{from } \mu^- \\
10^{77} & \text{from } K^- \\
10^{78} & \text{from } n^- \\
10^{79} & \text{from } \pi^- \\
10^{80} & \text{from } \mu^- \\
10^{81} & \text{from } K^- \\
10^{82} & \text{from } n^- \\
10^{83} & \text{from } \pi^- \\
10^{84} & \text{from } \mu^- \\
10^{85} & \text{from } K^- \
\end{cases}
\]

from [PB, Hümmer, and Winter, Phys. Rev. D83, 067303 (2011)]
Spectral features

$$\pi^+ \rightarrow \mu^+ + \nu_\mu,$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu,$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu,$$
$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu,$$

$$K^+ \rightarrow \mu^+ + \nu_\mu,$$

$$n \rightarrow p + e^- + \bar{\nu}_e.$$


$$\nu_\mu$$ flux after flavor mixing

from [PB, Hümmern, and Winter, Phys. Rev. D83, 067303 (2011)]
How the spectrum changes...

At example of GRB080603A:

1. Correction to analytical model (IC-FC → RFC)
2. Change due to full numerical calculation

from [Hümer, PB, and Winter, Phys. Rev. Lett. 108, 231101 (2012)]
Aggregation of fluxes

- Diffuse flux = result of large number of (unresolved) individual sources
- GRB rate follows SFR
- $z$ decoupled from other parameters
- General scaling: $F \propto d_L^{-2}$

From [PB, Hümmer, and Winter, Astropart. Phys. 35, 508 (2012)]
Re-computation of IC40 analysis

The new prediction

IceCube-40 GRB sample

Experimental realization:

- 117 observed bursts, between April 2008 and May 2009
- Individual neutrino spectra stacked
- Parameters measured or standard value
- Diffuse limit for 667 bursts per year

Result of re-computation

- Significantly reduced prediction
- Uncertainties of astrophysical parameters; tested ranges:
  - $t_v = 0.001 - 0.1 \text{ s}$,
  - $\Gamma = 200 - 500$,
  - $\alpha_p = 1.8 - 2.2$, and
  - $\epsilon_B/\epsilon_e = 0.1 - 10$
- Conservative bounds only with known parameters, here: known redshifts
- Additional uncertainty from statistics in stacking analysis

from [Hümmер, PB, and Winter, Phys. Rev. Lett. 108, 231101 (2012)]
Uncertainties for different computations

- **IceCube**: based on [Guetta, Spada, and Waxman, Astrophys. J. 559, 101 (2001)]: origin of target photons fixed (synchrotron, IC scattering)

- **Hümmel et al.**: uncertainties of parameters in basic fireball model, target photon spectrum from observation

- **He et al.**: most general assumption with emission radius as free parameter; includes possibilities of other models (dissipative photosphere, magnetic reconnection...)

One neutrino per cosmic ray

\[ p + \gamma \rightarrow \Delta^+ \rightarrow n + \pi^+ \]

\( n \): not subject of magnetical confinement and can contribute to cosmic rays

\[ n \rightarrow p e^- \bar{\nu}_e \]

\( \pi^+ \): contributes to neutrino flux

\[ \pi^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \nu_\mu \]

\[ \Rightarrow \] one \( \nu_\mu \) per CR proton
Observational bounds

Recent calculations exclude this (arbitrary) **one-to-one correspondence** for generic $\nu$-spectra.

Possible escape regimes

Different escape possibilities:

- **direct escape**: protons at highest energies escape directly.
- **optically thin neutron escape**: $n$ from $p\gamma$ interactions give usual contribution to CR, usually considered case.
- **optically thick neutron escape**: amount of $\nu$ enhanced due to higher number of interactions.

Standard escape may be constrained to limited region in parameter space.

[P.B, Spector, Waxman, and Winter, in preparation]
The cosmic ray connection
The “ultimate” goal

A selfconsistent multi-messenger picture

GRB models
Number of bursts per year, baryonic loading $f_e, f_\pi$
Fluence of burst

$\gamma$

Diffuse vs. point source data
Number of bursts per year, baryonic loading $f_e$

$\nu$

$\text{CR}$

Strong constraints

[Ahlers, Gonzalez-Garcia, and Halzen, Astropart. Phys. 35 (2), 87-94 (2011)]:
direct connection ($Y_\nu \approx Y_{\pi^+}$)

Contradictory claims in literature
Theoretical framework

Very long lifetimes $\kappa_i^{-1}$ of neutrino mass eigenstates $\nu_i (i = 1, 2, 3)$ may be tested by cosmological $\nu$s.

$$\kappa_i^{-1} \left[ \frac{s}{eV} \right] \equiv \frac{\tau_i [s]}{m_i [eV]} \simeq 10^2 \frac{L [\text{Mpc}]}{E [\text{TeV}]}$$

Proper differential decay equation including redshift dependence:

$$\frac{dN_i(E_0, z)}{dz} = -\kappa_i \frac{dL}{E_0} \frac{N_i(E_0, z)}{1 + z}$$

Correct distance measure for decay given by light-travel (or look-back) distance.

[PB, Bustamante, and Winter, JCAP 1210, 020 (2012)]
Theoretical framework

Very long lifetimes $\kappa_i^{-1}$ of neutrino mass eigenstates $\nu_i (i = 1, 2, 3)$ may be tested by cosmological $\nu$s.

$$\kappa_i^{-1} \left[ \frac{s}{eV} \right] \equiv \frac{\tau_i [s]}{m_i [eV]} \simeq 10^2 \frac{L [\text{Mpc}]}{E [\text{TeV}]}$$

Proper differential decay equation including redshift dependence:

$$\frac{dN_i(E_0, z)}{dz} = -\frac{\kappa_i}{E_0} \frac{dL}{dz} \frac{N_i(E_0, z)}{1 + z}$$

Correct distance measure for decay given by light-travel (or look-back) distance.

[PB, Bustamante, and Winter, JCAP 1210, 020 (2012)]

Observational bounds

Detection of neutrinos from SN1987A gives strong bound on lifetime of $\nu_1$ ($\kappa_1^{-1} \sim 10^4 \text{s eV}^{-1}$), and effectively $\nu_e$. 
Consequences of decay

Possible decay scenario:

- $\nu_2$ and $\nu_3$ could be unstable; $\nu_1$ bounded by SN1987A (stable)
- leads to suppression of cosmical $\nu_\mu$ and $\nu_\tau$
- $\nu_e$ nearly unaffected

$\Rightarrow$ currently only $\nu_e$ cascades give reliable bounds

[PB, Bustamante, and Winter, JCAP 1210, 020 (2012)]
Summary

- Standard GRB neutrino flux shape is modified by full particle physics treatment.

- Simulation of diffuse neutrino flux is model-dependent.

- Numerical re-analysis of IC40 GRB neutrino prediction is significantly lower than original prediction.

- Cosmic rays can be obtained from same calculation and constrain neutrino predictions. But there are possibilities to circumvent these.

- Neutrino decay could also reduce flux - apart from $\nu_1$ due to SN1987A bounds. $\nu_e$ cascades currently most reliable information.

- Ultimate goal: Selfconsistent particle physics model of astrophysical sources.
Back-up slides
Used inputs

- Proton spectrum $N'_p(E') \propto E'^{-2}$
- Photon spectrum resembling (simplified) Band function with parameters $\alpha, \beta,$ and $E_{\gamma, \text{break}}$
- Contributions to $\sigma_{p\gamma}$ from $\Delta(1232)$-resonance, higher resonances, $t$-channel (direct production), and high energy processes (multiple $\pi$)
- Decays of $\pi^\pm$, $\mu^\pm$, $n$, and $K^+$ considered, in case of $\mu^\pm$ including helicity dependence
  

- Simple neutrino mixing including three mass eigenstates and $\theta_{13} = 0$
- Normalization at the end matched to WB GRB $\nu_\mu$ flux bound
Revised fireball model

Corrections to shape \((c_s)\):

- Revised shape
- Correct energy losses of secondaries
- Full energy dependencies

See also \([\text{Li, Phys. Rev. D}85, 027301 (2012)]\).

Corrections to \(f_{\pi} \ (c_{f_{\pi}})\):

- Integral normalization of photon spectrum
- Rounding errors
- Width of \(\Delta\)-resonance

Compare to \([\text{Guetta et al., Astropart. Phys. 20, 429 (2004)]}\).

GRB rate from corrected SFR by Hopkins and Beacom

SFR by Hopkins and Beacom:

\[ \dot{\rho}_* \propto \begin{cases} 
(1 + z)^{3.44} & \text{for } 0 < z \leq 0.97 \\
10^{1.09} \cdot (1 + z)^{-0.26} & \text{for } 0.97 < z \leq 4.48 \\
10^{6.66} \cdot (1 + z)^{-7.8} & \text{for } 4.48 < z 
\end{cases} \]


Correction factor taken from

\[ \mathcal{E}(z) \propto (1 + z)^{1.2} \]
Single burst detection probability estimate

**Assumption:** Diffuse flux from 10000 bursts is at the level of the WB flux (1/10 of WB), leading to a certain number of events in the full IceCube detector during 10 years of operation.

**Question:** How near would a single burst have to be to lead to at least 3 events?

**Answer:** Burst has to be at most at $z_{\text{max}} = 0.14$ ($z_{\text{max}} = 0.05$). The probability to have at least one such a close burst is about 40% (2%).

from [PB, Hümmer, and Winter, Astropart. Phys. 35, 508 (2012)]
The measured cross section

\[ \sigma(\varepsilon) \text{ [\mu barn]} \]

\[ \varepsilon_r [\text{GeV}] \]

from [Hümmert, Rüger, Spanier, and Winter, Astrophys. J. 721, 630 (2010)],

Resulting aggregated flux

- Peak contribution from bursts at $z \lesssim 1$, few close bursts dominate
- Leads to shift to higher energies compared to single burst (with assumed $z = 2$)
- Characteristic features of single burst flux shape are preserved
- Analysis of other source parameters showed that only the magnetic field can wash out these features
Result of statistical analysis

Simulated 100,000 samples of $n$ bursts with different redshift, and extrapolated flux limits from stacked flux. Analysis of resulting limits gave the results:

<table>
<thead>
<tr>
<th>$n$</th>
<th>Rel. error 90%</th>
<th>Rel. error 3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.53 – 1.57</td>
<td>0.39 – 8.78</td>
</tr>
<tr>
<td>1000</td>
<td>0.72 – 1.25</td>
<td>0.64 – 5.15</td>
</tr>
<tr>
<td>10000</td>
<td>0.83 – 1.08</td>
<td>0.78 – 2.62</td>
</tr>
</tbody>
</table>

- Variation of extrapolated bound for 100 bursts (comparable with the 117 bursts from IceCube) still too high to rule out the (most basic) fireball model - due to statistics.
- For variation of multiple parameters effect gets even stronger.
- However, additional effects, such as bias through minimal observable flux when considering simultaneous variation of redshift and luminosity, reduce uncertainty compared to independent variation case.
Picked 100 000 samples with different redshifts according to redshift distribution shown earlier for each sample size.

<table>
<thead>
<tr>
<th>n</th>
<th>±10%</th>
<th>±20%</th>
<th>±50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.17</td>
<td>0.37</td>
<td>0.91</td>
</tr>
<tr>
<td>1000</td>
<td>0.30</td>
<td>0.69</td>
<td>0.98</td>
</tr>
<tr>
<td>10000</td>
<td>0.48</td>
<td>0.97</td>
<td>0.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n</th>
<th>Rel. error 90%</th>
<th>Rel. error 3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.53 – 1.57</td>
<td>0.39 – 8.78</td>
</tr>
<tr>
<td>1000</td>
<td>0.72 – 1.25</td>
<td>0.64 – 5.15</td>
</tr>
<tr>
<td>10000</td>
<td>0.83 – 1.08</td>
<td>0.78 – 2.62</td>
</tr>
</tbody>
</table>