PROTON DECAY THEORY REVIEW

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Introduction

STANDARD MODEL:

- renormalizable level: accidental B and L conservation (no invariants that violate B and/or L)
- non-perturbative level: $B+L$ violated by anomalies ('t Hooft), but the effect is nowadays negligible (possibly very important in the early universe - sphalerons crucial in leptogenesis)
- with higher-dimensional (non-renormalizable) operators one can violate B and/or L, but there is no guide how large are these operators.

Only physics beyond the SM can connect such operators with other physical phenomena and thus be testable.

The best such candidate is grand unification. In the ideal case it connects proton decay with fermion masses and mixings. So fitting the latter determines the former. Reality is of course less ideal.

I will consider only (predictive) GUTs in trying to get some theoretical input for nucleon decay.

Effective field theory analysis

The lowest dimensional operators that describe nucleon decay are of the form (schematically)

$$\mathcal{L}_{d=6} = \frac{\kappa}{M^2} qqql + h.c.$$

Weinberg, 79

Wilczek, Zee, 79

$q \ldots$ left-handed (Q) or right-handed (u_R, d_R) quark

$l \ldots$ left-handed (L) or right-handed (e_R) lepton

More precisely:

$$\begin{aligned}\mathcal{L}_{d=6} &= \frac{\kappa^{(1)}}{M^2}(QQ)(QL)\\ &+ \frac{\kappa^{(2)}}{M^2}(QQ)(u_R e_R)\\ &+ \frac{\kappa^{(3)}}{M^2}(u_R d_R)(QL)\\ &+ \frac{\kappa^{(4)}}{M^2}(u_R d_R)(u_R e_R) + h.c.\end{aligned}$$

All $d = 6$ (4-fermion) B or L violating operators can be written with these four. All of them are accidentally $(B - L)-$preserving.

They follow just from gauge symmetry and particle content of SM.

At this leading order:

- $B - L$ is a good (accidental) symmetry

  → no decays like $p(n) \to \nu(e^-)+$ meson
- final s-quark cannot appear with l^+, so processes like

$$p \to \overline{K^0} e^+ \qquad \text{or} \qquad n \to K^- e^+$$

  are suppressed

Since $M = M_{GUT} \gtrsim 10^{15-16}$ GeV, higher dimensional operators $\mathcal{O}_{d=7}/M^3$ even more suppressed.

Would finding these decay modes point toward lower scales, i.e. nucleon decay originating from non grand unification physics?

Not really!

- a final s quark plus l^+ is possible by a W exchange: suppressed, but not really forbidden
- $\Delta(B-L)=-2$ but $(B+L)$-preserving proton decay points towards $d=7$ operator, for example

$$\frac{c}{M^3}\mathcal{O}_{d=7}=\frac{c}{M^3}d^c d^c u^c L H$$

could have

1. non-GUT origins, for example R-parity violating susy

Vissani, '95

2. or GUT origins with intermediate scale M_I

$$\frac{1}{M^3}\to\frac{1}{M_{GUT}}\frac{1}{M_I^2}$$

Barr, Calmet, '12

Babu, Mohapatra, '12

Non-negligible $d = 7$ modes resurrect GUT baryogenesis:

these operators are $(B-L)$-violating, so they will not get washed-out by sphalerons

serious competition with leptogenesis

Babu, Mohapatra, '12

Grand unified symmetries only

In the literature you find that typical model has symmetry

$$G \times G' \times Z$$

$G \ldots$ a GUT group like SU(5) or SO(10)

$G' \ldots$ an Abelian or non-Abelian gauge or global symmetry (sometimes connected with flavor)

$Z \ldots$ a (product of) discrete symmetries (for ex. R-parity in susy)

Many possibilities on the market, for example:

$SO(10)\times$ textures

Babu, Pati, Wilczek, '98

$SO(10)\times U(1)\times D_3$

Dermíšek and Raby, '99

$SO(10)\times U(2)\times U(1)\times\ldots$

Blažek, Raby, Tobe, '99

$SO(10)\times U(1)\times Z_2\times Z_2$

Albright, Barr, 98, '00

$SO(10)\times U(1)\times Z_2$

Babu, Pati, Tavartkiladze, '10

$\ldots$

Several nice models do nor have good theoretical or experimental motivations, results depend on specific assumptions

Assuming ad-hoc symmetries is dangerous: it can cancel the effect you are interested in and want to measure

For example (extreme case): imagine a model with symmetries

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_B \times U(1)_L$

It predicts (perturbative) proton stability. Would you take its prediction seriously?

→ stick to minimalist's approach: no extra symmetry on top of grand unification (G only) and eventually supersymmetry. It is restrictive, but it is what we have to check first, before more data are coming

Non-supersymmetric GUTs

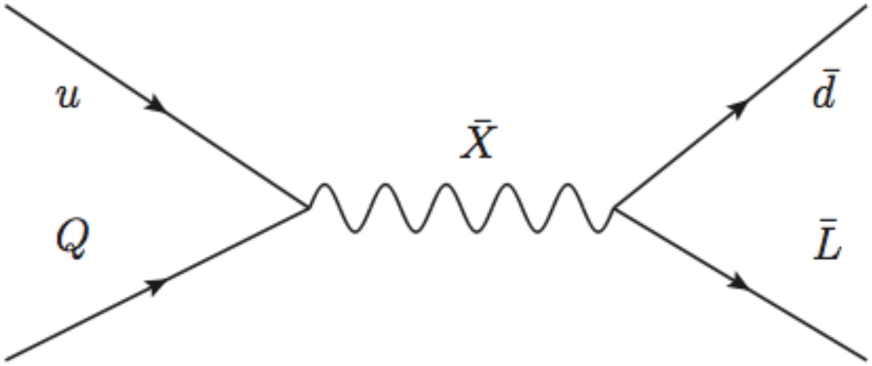
In SU(5) all fermions of one generations live in two representations

$$5_F = d_R(3) + L(2) \quad , \quad 10_F = u_R(3) + Q(6) + e_R(1)$$

There are 24 SU(5) gauge bosons

$$24_V = gluons(8) + W^{\pm}(2) + Z(1) + \gamma(1) + X(6) + \bar{X}(6)$$

$X(3, 2, -5/6)$, $\bar{X}(\bar{3}, 2, 5/6)$ gauge bosons have mass M_{GUT} (where three SM gauge couplings unify) and mediate proton decay



$$\text{amplitude} \propto \frac{1}{M_{GUT}^2} \rightarrow \tau \propto M_{GUT}^4$$

Strong dependence on M_{GUT}. Unification constraints:

$$\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT})$$

This depends on the particle spectrum between M_Z and M_{GUT}

Unification constraints $\rightarrow$ M_{GUT} and particle spectrum (if not too many degrees of freedom)

The original minimal Georgi-Glashow SU(5) model

$$3 \times (10_F + \bar{5}_F) + 5_H + 24_H + 24_V$$

does not work, because

- SM gauge couplings do not unify
- neutrino mass vanishes

One can add different representations to make it realistic

Two examples:

1. add 15_H

Doršner, Fileviez Perez, 05

Doršner, Fileviez Perez, Gonzalez Felipe, 05

- parts of 15_H (not necessarily very light) modify RGE's to unify
- neutrino mass from type II seesaw: $(1,3,1) \subset 15_H$
- $M_{GUT} \approx 10^{14}$ GeV (cancellations due to Yukawas)
- proton decay fast: to be discovered soon

2. add 24_F

BB, Senjanović, '06; BB, Nemevšek, Senjanović, '07

- new states in 24_F contribute to RGE's:

$$\begin{aligned} 24 &= O(8,1,0)+T(1,3,0)+S(1,1,0) \\ &+ X(3,2,-5/6)+\bar{X}(\bar{3},2,5/6) \end{aligned}$$

- unification occurs iff

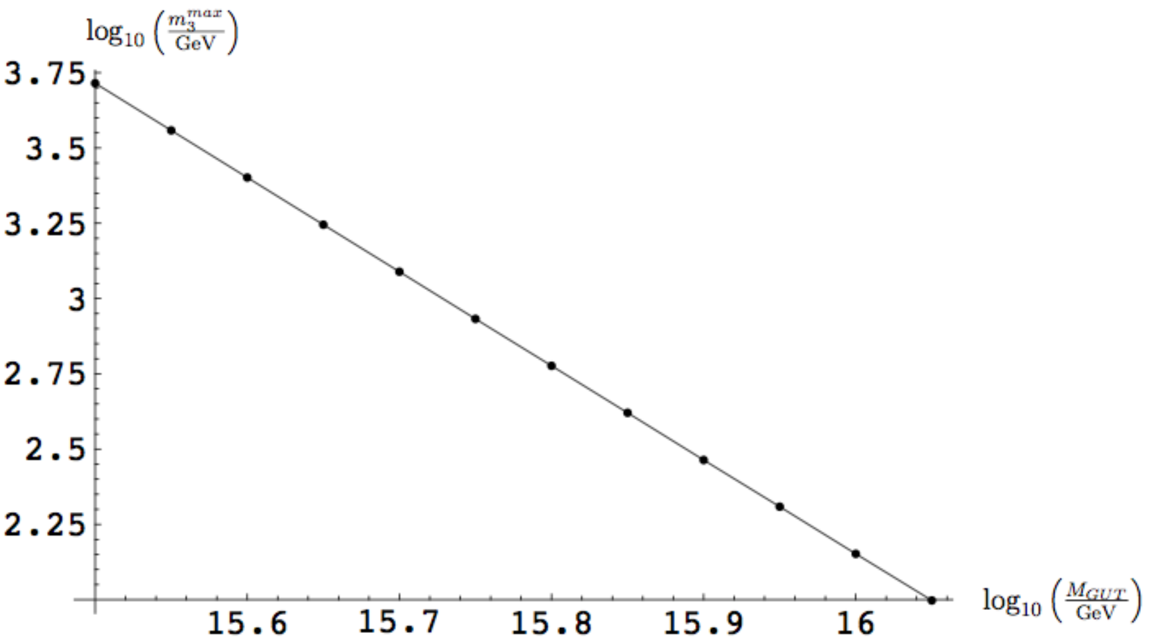
$$m_T \approx 10^3 \text{ GeV} \;\; , \;\; m_O \approx 10^8 \text{ GeV} \;\; , \;\; m_{X,\bar{X}} \approx 10^{13} \text{ GeV}$$

→ prediction of a light (TeV) weak fermionic triplet

- neutrino mass from mixed type I (S) and III (T) see-saw
- same Yukawa that describe neutrino mass appear in triplet decay (to check at LHC)

- proton decay faster if triplet heavier

$m_3^{max} - M_{GUT}$ at two loops



large $m_T \approx 5$ TeV $\rightarrow M_{GUT} \approx 10^{15.5}GeV\rightarrow \tau_p \approx 10^{34}$yrs

small $m_T \approx 100$ GeV $\rightarrow M_{GUT} \lesssim 10^{16}GeV\rightarrow \tau_p \lesssim 10^{36}$yrs

We have already some preliminary data:

CMS (2012) at 95% CL: $m_T \gtrsim 180 - 200$ GeV (depending on the Yukawas - triplet branching ratios)

If LHC does not find the triplet:

$m_T \gtrsim 700$ GeV $\rightarrow \tau_p \lesssim 10^{35}$ yrs (or the model is ruled out)

Summary of ordinary (non-supersymmetric) models:

- $\tau_p \propto M_{GUT}^4$
- main decay mode typically $p \to \pi^0 e^+$
- in minimal realistic models $\tau_p \lesssim 10^{36}$ yrs

Supersymmetric GUTs

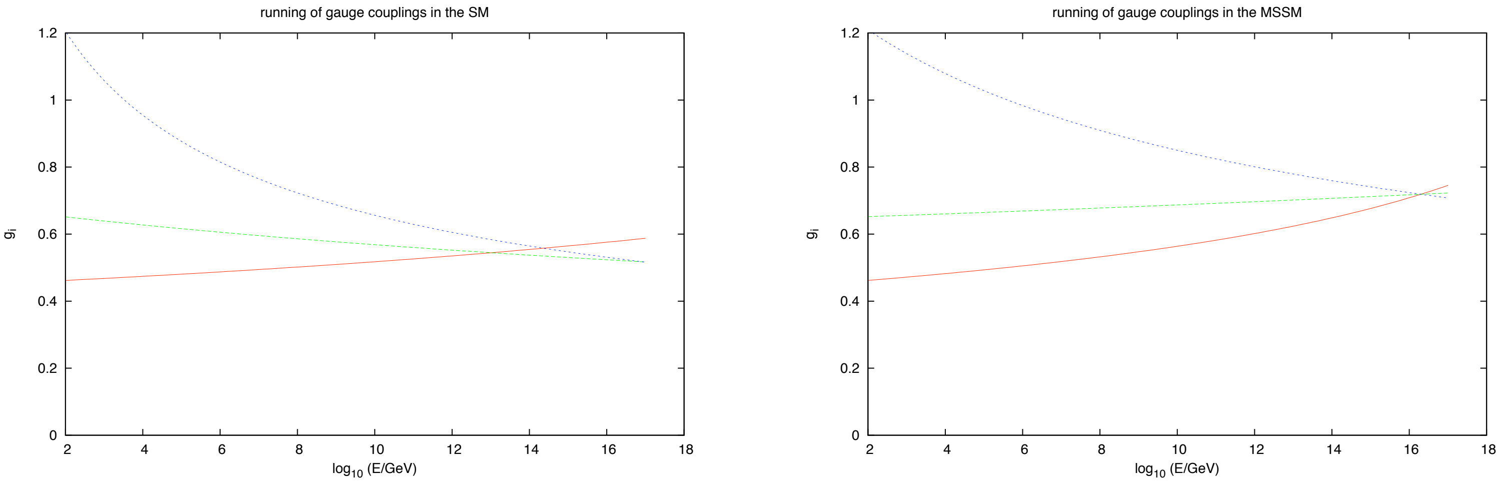
Three big extra uncertainties here:

- R-parity violation → makes susy GUTs quite non-predictive for p-decay (unless R-parity predicted, like in SO(10) with 126). Usually assumed to vanish.
- $1/M_{Planck}$ suppressed $B + L$ violating operators (unless they are forbidden, like in E_6 GUTs)
- susy breaking terms

The first two contributions typically mediate a too large proton decay rate

→ we can give in many cases at most upper limits to proton decay lifetime, assuming no further cancellation occur

In contrast to what happens in non-susy theories, here the low energy MSSM spectrum already reaches unification of couplings



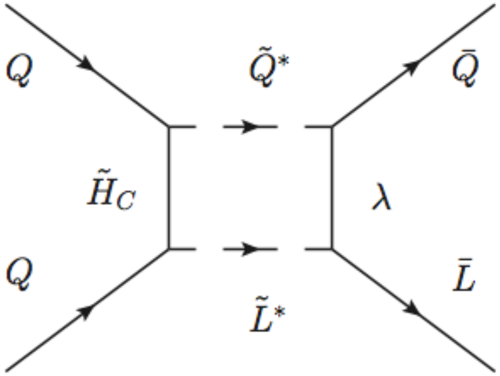
Dimopoulos, Raby, Wilczek, '81; Ibañez, Ross, '81

Einhorn, Jones, '82; Marciano, Senjanović, '82

- good, a special reason to believe in supersymmetric unification
- bad, no strong constraint on theories (but sometimes important)

The rate for nucleon decay in low-energy susy models is typically dominated by $d = 5$ operators (schematically)

$$\tau^{-1} \approx \left| \left(\frac{Y^2}{M_C} \right) \left(\frac{\alpha}{4\pi} \frac{m_\lambda}{m_{\tilde{f}}^2} \right) \right|^2 m_p^5$$



Weinberg, '82; Sakai, Yanagida, '82

Hisano, Murayama, Yanagida, '92 ; Lucas, Raby, '96; Goto, Nihei, '98

$Y^2 \ldots$ product of two Yukawa couplings

$M_C \ldots$ color triplet mass

$(\alpha \ldots)$ MSSM loop factor (λ gaugino or higgsino, $\tilde{f}$ sfermion)

$m_p^5 \ldots$ from strong QCD dynamics (lattice)

Lifetime strongly model dependent

Renormalizable sugra SU(5)

$(3 \times (10_F + \overline{5}_f) + 5_H + \overline{5}_H + 24_H + 24_V)$:

- $Y^2 = Y_U Y_D (= Y_U Y_E)$
- sfermion mixing $\propto$ fermion mixing
- LHC-friendly values for soft parameters: $m_\lambda \approx m_{\tilde{f}} \lesssim \mathcal{O}(\text{TeV})$
- $M_C \approx 10^{14-15}$ GeV from RGE constraints (2-loop unification)

$$\tau \approx 10^{29} \text{ years}$$

Murayama, Pierce, '01

Too fast, since $\tau_{exp}(p \to K^+ \bar{\nu}) \gtrsim 4 \cdot 10^{33}$ yrs

However no real reason for all these assumptions.

The minimal renormalizable sugra SU(5) is anyway ruled out by bad fermion mass relations ($M_D = M_E$). Different cures:

- non-renormalizable operators ($M_{GUT}/M_{Pl} \gtrsim 10^{-3} - 10^{-2}$ not that small)
- extra representations ($5_F + \overline{5}_F$, $45_H + \overline{45}_H$, etc)
- MSSM threshold corrections

With these corrections proton decay can change drastically.

1. Non-renormalizable operators

- Yukawas Y^2 not necessarily the low-energy ones

  Ellis, Gaillard, '79; Emmanuel-Costa, Wiesenfeldt, '03

- Particle spectrum arbitrary. If $T = (1,3,0) \subset 24$ and $O = (8,1,0) \subset 24$ lighter than M_{GUT}
  $\rightarrow$ RGE change and M_{GUT} and M_C could increase :

$$M_{GUT}^{new} = M_{GUT} \left(\frac{M_{GUT}}{\sqrt{m_T m_O}} \right)^{1/2} \quad , \quad M_C^{new} = M_C \left(\frac{m_T}{m_O} \right)^{5/2}$$

  Bachas, Fabre, Yanagida, '96; Chkareuli, Gogoladze, '98

- sfermion mixings $\neq$ fermion mixings

  BB, Fileviez Perez, Senjanović, '02

→ no real theoretical limit on proton lifetime

2. extra vectorlike $5_F + \overline{5}_F$:

   - weak doublets from extra $5_F + \overline{5}_F$ correct the wrong relation $Y_D = Y_E$

   - color triplets from extra $5_F + \overline{5}_F$ are lighter to satisfy RGE constraint on unification, while color triplet from $5_H + \overline{5}_H$ mediate nucleon decay

   $\rightarrow$ theoretical proton lifetime longer than experimental

   at least one p decay mode not slower than $2 \cdot 10^{34}$ yrs

   providing all spartners lighter than 3 TeV

Babu, BB, Tavartkiladze, '12

decay mode	$\tau_{d=5}$ (yrs)	exp. lower limit (90 % CL)
$p \to \bar{\nu} K^+$	$2 \cdot 10^{34}$	$4 \cdot 10^{33}$
$p \to \mu^+ K^0$	$4 \cdot 10^{34}$	$1 \cdot 10^{33}$
$p \to \mu^+ \pi^0$	$7 \cdot 10^{34}$	$1 \cdot 10^{34}$
$p \to \bar{\nu} \pi^+$	$3 \cdot 10^{34}$	$4 \cdot 10^{32}$
$n \to \bar{\nu} K^0$	$8 \cdot 10^{33}$	$1 \cdot 10^{32}$
$n \to \bar{\nu} \pi^0$	$6 \cdot 10^{34}$	$1 \cdot 10^{33}$

But of course, heavier sfermion mass can easily make lifetime longer: $\tau \propto m_{\tilde{f}}^4$

None of above theoretical limits could probably be reached by Super-Kamiokande

SU(5) typically (with few exceptions) not attractive because of neutrino mass (like in SM)

SO(10) much better:

16_F = SM fermions + ν_R (prediction of right-handed neutrino)

Seesaw mechanism automatic, its scale connected with rank-breaking (SU$(2)_R$ and $B-L$) of SO(10)

Minimal supersymmetric SO(10)

Clark, Kuo, Nakagawa, '82; Aulakh, Mohapatra, '83

Aulakh, BB, Melfo, Senjanović, Vissani, '03

1. The model:
   - to break rank and predict R-parity use $126_H + \overline{126}_H$

     Aulakh, Benakli, Senjanović, '97

     Aulakh, Melfo, Senjanović, '98

     Aulakh, Melfo, Rašin, Senjanović, '99
   - to get main contribution to fermion masses use 10_H
   - to break SO(10) and correct bad mass relations use 210_H

     Babu, Mohapatra, '92

2. under assumption of renormalizability one derives

BB, Doršner, Nemevšek, '08

- split susy scenario, $m_\lambda \approx 100$ TeV, $m_{\tilde{f}} \approx 10^{14}$ GeV
  → no $d = 5$ p-decay modes, no uncertainties with soft terms, no MSSM threshold corrections to fermion masses
- borderline $d = 6$ p-decay mode: $\tau(p \to \pi^0 e^+) \lesssim 1.2 \cdot 10^{34}$ yrs;
  $BR(p \to \pi^+ \bar{\nu}) = 0.49$, $BR(p \to \pi^0 e^+) = 0.44$,
  $BR(p \to K^0 \mu^+) = 0.05$
- good fit of fermion masses (at that time prediction $\theta^l_{13} \gtrsim 0.1$)

Goh, Ng, Mohapatra, '03; Bertolini, Malinsky, '04;
Babu, Macesanu, '04; Bertolini, Malinsky, Schwetz, '05

3. Warning: assumption of renormalizability very strong. There is a cutoff in this theory (blow-up of couplings) already at $\Lambda/M_{GUT} \approx 10$

   Corrections should be included, and they can be large

Summary for supersymmetric models:

- large uncertainty due to heavy (color triplet) and light (sfermion and gaugino-higgsino) spectrum: $\tau \propto M_C^2 m_{\tilde{f}}^4 / m_\lambda^2$
- large uncertainty due to RPV, M_{Planck} or cutoff-suppressed operators
- LHC data could partially improve the situation
- proton lifetimes in susy GUTs are at best upper limit

Another possible uncertainty in models with large representations ($126 + \overline{126}$, 210, etc):

Dixit, Sher, '89

typically large finite 1-loop GUT threshold corrections

Weinberg, '79; Hall, '81

to get them one needs to know exactly a (typically) hardly calculable heavy spectrum

Wright, '94

the only explicit example so far shows a strong suppression of $d = 5$ p-decay

Aulakh, '11

If this confirmed (and shown to remain perturbative), it would mean proton decay problem with large representations relaxed

However caution because of perturbativity issues: at large N

$$\lambda^2 \to N\lambda^2$$

If the effect large most probably perturbativity lost.

Conclusions

Several hints that grand unification is around. These are good theories of proton decay and fermion masses.

I mentioned here only some models that are predictive, not necessarily models that are likely to be confirmed. No extra symmetries assumed, only unified gauge symmetry

Predictive non-supersymmetric models could be tested in the near future in combined proton decay experiment + LHC

With orders of magnitude (and superficial) estimates susy models give dangerously fast p-decay. More realistic estimates less restrictive. Uncertainties still very large.

BACK-UP MATERIAL

There are 24 SU(5) gauge bosons

$$\begin{pmatrix} gluons(8) & X(6) \\ \bar{X}(6) & W^{\pm}, Z, \gamma(4) \end{pmatrix}$$

$X(3,2,-5/6)$, $\bar{X}(\bar{3},2,5/6)$ gauge bosons have mass M_{GUT} (where three SM gauge couplings unify) and mediate proton decay

Grand unification

The SM has some theoretical issues

- mysterious quantization of electric charge
- 3 different gauge groups (and couplings)
- 5 different representations (15 d.o.f.) per generation
- 4 different uncorrelated Yukawa matrices

and experimental shortcomings

- unclear origin of neutrino mass (if Majorana)
- no dark matter candidate

Most of the above points can be solved or improved in grand unified theories (GUTs)

For example in minimal SU(5) or SO(10):

- charge quantization just follows from quantized values of non-abelian gauge group generators
- one single gauge group and gauge coupling at some large energy scale (of order 10^{16} GeV or so)
- 2 representations $10_F(= Q + u_R + e_R) + \bar{5}_F(= L + d_R)$ in SU(5) and only one $16_F(SM + \nu_R)$ in SO(10)
- typically less Yukawa matrices, for example 2 in renormalizable minimal supersymmetric SO(10)
- neutrino mass from see-saw mechanism in SO(10)
- existence of dark matter candidate not guaranteed but sometimes possible, model dependent
- a generic prediction of grand unification is nucleon instability