Leptonic mixing, CP violation and Leptogenesis

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based collaborations with:

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In the Standard Model neutrinos are strictly massless

- No Dirac mass since ν_R is not introduced
- No Majorana mass at tree level, since no scalar triplet is introduced.
- No Majorana mass at higher orders, due to exact B-L conservation

$\nu_L^T \, m \, C \, \nu_L$ violates (B-L) by 2 units

Same applies to SU(5) GUT, where B−L is an accidental symmetry

No leptonic mixing in the SM:

$$\bar{\nu}^0_{L_i} \gamma_\mu l^0_{L_i} W^\mu \qquad \nu^0_{L_i}, l^0_{L_i} \text{ weak eigenstates}$$

After diagonalization of the charged lepton mass matrix:

$$\bar{\nu}^0_L \gamma^\mu V l_L \qquad l_L \text{ mass eigenstates}$$

But V can be eliminated through a redefinition of ν^0_L:

$$\bar{\nu}_L \gamma_\mu l_L W^\mu \qquad \text{flavour diagonal}$$

Observation of neutrino oscillations provides clear evidence for New Physics beyond the SM.

Minimal extension of the SM which allows for non-vanishing neutrino masses:

A "strange" feature of the SM:

No ν_{R_i} are introduced.

What happens if we introduce ν_{R_i}?

Dirac masses for neutrinos are generated:

$$(Y_\nu)_{ij}\, \bar{L}_i\, \tilde{\phi}\, \nu_{R_j} \longrightarrow m_D = (Y_\nu)_{ij} \frac{v}{\sqrt{2}}$$

If one writes the most general Lagrangian consistent with gauge invariance and renormalizability, one has to include the mass term:

$$(M_R)_{ij}\, \nu_{R_i}^T C\, \nu_{R_j}$$

One may have $M_R \gg v$, since the mass term is gauge invariant.

This leads to the seesaw mechanism, with:

$$(m_\nu)_{light} \approx \frac{v^2}{M}$$

$$(M)_{heavy} \approx M_R$$

Note that this minimal extension of the SM, sometimes denoted SMν, is actually "simpler" and more "natural" than the SM, providing a simple and plausible explanation for the smallness of neutrino masses.

For the moment let us consider the low energy limit of the SMν.

Neutrino masses and mixing at low energies:

$$\mathcal{L}_{mass} = -\overline{l}_L \, m_\ell \, l_R - \frac{1}{2} \nu_L^T C \, m_\nu \, \nu_L + h.c.$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{l}_L \, \gamma^\mu \nu_L W_\mu + h.c.$$

m_ℓ, m_ν encode all information about lepton masses and mixing in general

m_ℓ — arbitrary complex matrix

m_ν — symmetric complex matrix

There is a great redundancy in m_ℓ, m_ν so not all of its physical parameters are physical. This redundancy stems from the freedom to make weak-basis (WB) transformations:

$$\nu_L = W_L \nu'_L \;;\; \ell_L = W_L \ell'_L \;;\; \ell_R = W_R \ell'_R$$

W_L, W_R unitary matrices

m_ℓ, m_ν transform as:

$$m'_\ell = W_L^\dagger m_\ell W_R \;;\; m'_\nu = W_L^T m_\nu W_L$$

One can use the freedom to make WB transformations to go to a basis where

$$m_\ell = d_\ell \longrightarrow \text{diagonal and real}$$

In this basis, one can still make a rephasing:

$$\ell''_{L,R} = K_L \, \ell'_{L,R} \quad ; \quad \nu''_L = K_L \, \nu'_L$$

with $K_L = \mathrm{diag}(e^{i\varphi_1}\ e^{i\varphi_2}\ e^{i\varphi_3})$. Under this rephasing d_ℓ remains invariant, but m_ν transforms as:

$$(m''_\nu)_{ij} = e^{i(\varphi_i + \varphi_j)} (m'_\nu)_{ij}$$

One can eliminate n phases from m_ν

So the number of physical phases in m_ν is:

$$\boxed{N_\phi = \frac{1}{2}n(n+1) - n = \frac{1}{2}n(n-1)}$$

For $n=3$ one has $N_\phi = 3$. So altogether one has in m_ν:

$$|(m_\nu)_{ij}| \longrightarrow 6 \text{ real parameters}$$
$$N_\phi \longrightarrow 3 \text{ phases}$$

The individual phases of $(m_\nu)_{ij}$ have no physical meaning because they are not rephasing invariant. But one can construct polynomials of $(m_\nu)_{ij}$ which are rephasing invariant

Examples of rephasing invariant polynomials:

$$P_1 \equiv (m_\nu^*)_{11} (m_\nu^*)_{22} (m_\nu)_{12}^2 \; ; \quad P_2 \equiv (m_\nu^*)_{11} (m_\nu^*)_{33} (m_\nu)_{13}^2$$

$$P_3 \equiv (m_\nu^*)_{33} (m_\nu^*)_{12} (m_\nu)_{13} (m_\nu)_{23}$$

Generation of leptonic mixing in the charged current

$$U_L^{\ell\dagger} m_\ell U_R^\ell = d_\ell \quad ; \quad U^{\nu T} m_\nu U^\nu = d_\nu$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma_\mu U \nu_L W^\mu + h.c.$$

$$U = U_L^{\ell\dagger} U^\nu \rightarrow \text{PMNS matrix}$$

In this basis, there is still freedom to rephase the charged lepton fields: $\ell_j \rightarrow \ell_j' = \exp(i\phi_j)\ell_j$

D) Due to the Majorana nature of neutrinos the rephasing:

$$\nu_k \to \nu_k' = \exp(-i\psi_k)\,\nu_k \;;\; \psi_k \text{ arbitrary}$$

is not allowed, since it would not leave the Majorana mass terms invariant: $\nu_{Lk}^T\, C\, m_k\, \nu_{Lk}$

In the mass eigenstate basis:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\left(\bar{e}\;\bar{\mu}\;\bar{\tau}\right)_L \gamma_\mu \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}_L W^\mu + \text{h.c.}$$

For the moment we do not introduce the contraints of 3×3 unitarity. Note that in the contest of type-one seesaw the PMNS matrix is not unitary.

Rephasing invariant quantities.

Recall the situation in the quark sector:

$$(\bar{u}\ \bar{c}\ \bar{t})_L \gamma_\mu \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}_L W^\mu + h.c.$$

Rephasing invariant quantities:
9 moduli
4 rephasing invariant phases

$$\beta \equiv \arg(-V_{cd} V_{tb} V_{cb}^* V_{td}^*)$$

$$\gamma \equiv \arg(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*)$$

$$\chi \equiv \beta_s = \arg(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$$

$$\chi' = \arg(-V_{us} V_{cd} V_{ud}^* V_{cs}^*)$$

Novel feature in the leptonic sector with Majorana neutrinos:

rephasing invariant bilinears of the type:

$$\arg\left(U_{\ell\alpha} U^*_{\ell\beta}\right)$$ (no summation of repeated indices)

"Majorana-type phases"

There are six independent Majorana-type phases. This is true even when unitarity is not imposed on U_{PMNS}. It applies to a general framework with an arbitrary number of right-handed neutrinos.

A possible choice for the six independent Majorana-type phases:

$$\beta_1 \equiv \arg\left(U_{e1} U_{e2}^*\right) \qquad \gamma_1 \equiv \arg\left(U_{e1} U_{e3}^*\right)$$

$$\beta_2 \equiv \arg\left(U_{\mu 1} U_{\mu 2}^*\right) \qquad \gamma_2 \equiv \arg\left(U_{\mu 1} U_{\mu 3}^*\right)$$

$$\beta_3 \equiv \arg\left(U_{\tau 1} U_{\tau 2}^*\right) \qquad \gamma_3 \equiv \arg\left(U_{\tau 1} U_{\tau 3}^*\right)$$

One can choose the following four independent Dirac-type invariant phases:

$$\sigma_{e\mu}^{12} \equiv \arg\left(U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*\right) = \beta_1 - \beta_2$$

$$\sigma_{e\tau}^{12} \equiv \arg\left(U_{e1} U_{\tau 2} U_{e2}^* U_{\tau 1}^*\right) = \beta_1 - \beta_3$$

$$\sigma_{e\mu}^{13} \equiv \arg\left(U_{e1} U_{\mu 3} U_{e3}^* U_{\mu 1}^*\right) = \gamma_1 - \gamma_2$$

$$\sigma_{e\tau}^{13} \equiv \arg\left(U_{e1} U_{\tau 3} U_{e3}^* U_{\tau 1}^*\right) = \gamma_1 - \gamma_3$$

A "Surprise":

If one assumes 3×3 unitarity of U_{PMNS}, the full leptonic mixing matrix can be obtained from the six independent Majorana phases.

- Normalization of rows and columns plays an important rôle. Prevents "blowing up" of unitarity triangles.

For three generations and assuming 3×3 unitarity the U_{PMNS} matrix can be parametrized by:

$$U = VK \qquad K = \mathrm{diag.}(1, e^{i\alpha_1/2}, e^{i\alpha_2/2})$$

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ \cdot & \cdot & s_{23}c_{13} \\ \cdot & \cdot & c_{23}c_{13} \end{bmatrix}$$

One can eliminate the phase δ from the first row by writing:

$$U = VK' \; ; \quad K' = \mathrm{diag}(1\; 1\; e^{i\delta})K$$

convenient for the analysis of $0\nu\beta\beta$.

Dirac and Majorana unitarity triangles

Dirac unitarity triangles

$$T_{e\mu} : U_{e1} U^*_{\mu 1} + U_{e2} U^*_{\mu 2} + U_{e3} U^*_{\mu 3} = 0$$

$$T_{e\tau} : U_{e1} U^*_{\tau 1} + U_{e2} U^*_{\tau 2} + U_{e3} U^*_{\tau 3} = 0$$

$$T_{\mu\tau} : U_{\mu 1} U^*_{\tau 1} + U_{\mu 2} U^*_{\tau 2} + U_{\mu 3} U^*_{\tau 3} = 0$$

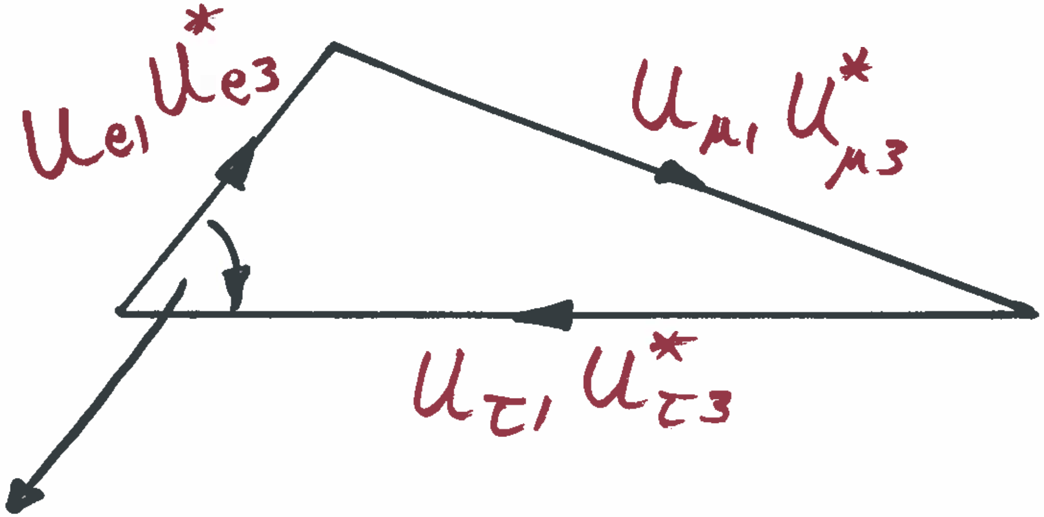
Majorana unitarity triangles

$$T_{12} : U_{e1} U^*_{e2} + U_{\mu 1} U^*_{\mu 2} + U_{\tau 1} U^*_{\tau 2} = 0$$

$$T_{13} : U_{e1} U^*_{e3} + U_{\mu 1} U^*_{\mu 3} + U_{\tau 1} U^*_{\tau 3} = 0$$

$$T_{23} : U_{e2} U^*_{e3} + U_{\mu 2} U^*_{\mu 3} + U_{\tau 2} U^*_{\tau 3} = 0$$

Example of a Majorana Triangle



$$\arg\left(-U_{e1} U_{e3}^* U_{\tau 1}^* U_{\tau 3}\right) = \pi - (\delta_3 - \delta_1) \rightarrow \text{Dirac-type phase}$$

Majorana phases – they give the directions of the sides of the Majorana unitary triangles. "Arrows have no meaning! They can be reversed if one makes, for example the rephasing: $\nu_3 \rightarrow \nu_3' = -\nu_3$.

The limit of CP invariance

Majorana triangles provide necessary and sufficient conditions for having CP invariance with Majorana neutrinos.

- Vanishing of their common area:

$$A = \frac{1}{2} |\mathrm{Im}\, Q|$$

- Orientation of all the "collapsed" triangles along the real axis or the imaginary axis. If one of these triangles T_{ik} is parallel to the imaginary axis, that means that the neutrinos i, k have opposite CP parities.

Neutrinoless double-beta decay ($0\nu\beta\beta$)

$0\nu\beta\beta$ is sensitive to Majorana-type phases:

$$|m_{ee}|^2 = m_1^2 |U_{e1}|^4 + m_2^2 |U_{e2}|^4 + m_3^2 |U_{e3}|^4 +$$

$$+ 2 m_1 m_2 |U_{e1}|^2 |U_{e2}|^2 \cos(2\beta_1) + 2 m_1 m_3 |U_{e1}|^2 |U_{e3}|^2 \cos(2\gamma_1)$$

$$+ 2 m_2 m_3 |U_{e2}|^2 |U_{e3}|^2 \cos[2(\beta_1 - \gamma_1)]$$

Note that the angle $(\gamma_1 - \beta_1)$ is the argument of $(U_{e1}^* U_{e2} U_{e1} U_{e3}^*)$ which is not a rephasing invariant Dirac-type quartet.

If one adopts an explicit parametrization:

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ \times & \times & s_{23}c_{13}e^{i\delta} \\ \times & \times & c_{23}c_{13}e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 & & \\ & e^{i\alpha_1/2} & \\ & & e^{i\alpha_2/2} \end{bmatrix}$$

$$m_{ee} = \left| \left[c_{13}^2 \left(m_1 c_{12}^2 + m_2 e^{-i\alpha_1} s_{12}^2 \right) + m_3 e^{-i\alpha_2} s_{13}^2 \right] \right|$$

This is the reason why this parametrization is useful for the analysis of $0\nu\beta\beta$.

Determining the neutrino mass matrix from experiment

We have seen that in the WB where the charged lepton mass matrix is diagonal, real one has:

$$m_\ell = \text{diag.}(m_e, m_\mu, m_\tau) \; ; \qquad \mathcal{M}_\nu = \begin{bmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ \cdot & m_{\mu\mu} & m_{\mu\tau} \\ \cdot & \cdot & m_{\tau\tau} \end{bmatrix}$$

Can use rephasing freedom to make $m_{ee}, m_{\mu\mu}, m_{\tau\tau}$ real, but $m_{e\mu}, m_{e\tau}, m_{\mu\tau}$ complex.

Altogether: 6 real parameters + 3 phases = 9

How many physical quantities in M_ν can be obtained through feasible experiments?

Δm^2_{12}, Δm^2_{23}

θ_{12}, θ_{23}, θ_{13}

$\mathrm{Im}\, Q \to$ CP violation in ν oscillations

$0\nu\beta\beta \to m_{ee}$

$\Bigg\}$ 7 measurable quantities (Glashow's counting)

$\underbrace{7 < 9}$ → S. Glashow, Frampton, Mafartia

... We arrive at the dreadful conclusion that no presently conceivable set of feasible experiments can fully determine the neutrino mass matrix!

Some of the ways out:

- Postulate "texture zeros" in M_ν → S. Glashow et al
  various sets of zero allowed by experiment
- Postulate $\det. M_\nu = 0$ → G.C.B, R.G. Felipe, F. Joaquim, T. Yanagida
  7 parameters in M_ν!

CP-odd Weak-basis invariants in the leptonic sector with Majorana neutrinos

The relevant part of the Lagrangian is:

$$\mathcal{L} = -\bar{l}_L m_l l_R - \frac{1}{2} \nu_L^T C m_\nu \nu_L + \frac{g}{\sqrt{2}} \bar{l}_L \gamma_\mu \nu_L W^\mu + h.c.$$

- The CP transformation properties of the various fields are dictated by the part of the Lagrangian which conserves CP, namely the gauge interactions

- One has to keep in mind the fact that the gauge sector of the SM does not distinguish the various families of fermions. The most general CP transformations which leave $\mathcal{L}_{gauge}$ invariant is:

$$CP\, \ell_L (CP)^\dagger = W_L \gamma^0 C\, \bar{\ell}_L^{\,T}$$

$$CP\, \nu_L (CP)^\dagger = W_L \gamma^0 C\, \bar{\nu}_L^{\,T}$$

$$CP\, \ell_R (CP)^\dagger = W_R \gamma^0 C\, \bar{\ell}_R^{\,T}$$

where W_L, W_R are unitary matrices acting in generation space.

The Lagrangian of the leptonic sector conserves CP if and only if the leptonic mass matrices m_ν, m_ℓ satisfy:

$$W_L^T \, m_\nu \, W_L = - m_\nu^* \quad ; \quad W_L^\dagger \, m_\ell \, W_R = m_\ell^*$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$W_L^\dagger \, \tilde{h}_\nu \, W_L = (\tilde{h}_\nu)^* \qquad\qquad W_L^\dagger \, h_\ell \, W_L = h_\ell^*$$

$\tilde{h}_\nu \equiv h_\nu^*$

$$W_L^\dagger [\tilde{h}_\nu, h_\ell] W_L = [\tilde{h}_\nu^*, h_\ell^*] = [\tilde{h}_\nu^T, h_\ell^T] = -[\tilde{h}_\nu, h_\ell]^T$$

$$W_L^\dagger [\tilde{h}_\nu, h_\ell]^3 W_L = -\left\{ [\tilde{h}_\nu, h_\ell]^3 \right\}^T$$

CP invariance implies $\longrightarrow$ $\mathrm{tr}[\tilde{h}_\nu, h_\ell]^3 = 0$!!

CP invariance $\Rightarrow$ $\mathrm{tr}\left[\tilde{h}_\nu, h_\ell\right]^3 = 0$

Valid for an arbitrary number of generations!!

First derived for the quark sector by G.C.B, J.Bernabeu, M.Gronau

$$\mathrm{tr}\left[\tilde{h}_\nu, h_\ell\right]^3 = -6i\,(m_\mu^2 - m_e^2)(m_\tau^2 - m_\mu^2)(m_\tau^2 - m_e^2) \times \Delta m_{21}^2\, \Delta m_{31}^2\, \Delta m_{32}^2 \times \mathrm{Im}\, Q$$

$$\mathrm{Im}\, Q = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin\delta$$

This invariant is sensitive to Dirac type CP violation

For 3 generations, the vanishing of this invariant is the necessary and sufficient for the absence of Dirac-type CP violation.

What about Majorana-type CP violation?

Using the previous method, one can derive (G.C.B., L. Lavoura, M.N. Rebelo) that the following invariant is sensitive to Majorana type CP violation:

$$\boxed{I^{CP}_{\text{Majorana}} = \text{Im}\,\text{tr}\left(m_\ell m_\ell^\dagger m_\nu^* m_\nu m_\nu^* m_\ell^T m_\ell^* m_\nu\right)}$$

The simplest way to check that $I^{CP}_{\text{Maj.}}$ is sensitive to Majorana-type CP violation is by evaluating in the case of 2 generations of Majorana neutrinos:

$$I^{CP}_{\text{Maj.}} = \frac{1}{4} m_1 m_2 \, \Delta m^2_{21} \left(m_\mu^2 - m_e^2\right) \sin^2(2\theta) \sin 2\gamma$$

$$U = \begin{bmatrix} \cos\theta & -\sin\theta\, e^{i\gamma} \\ \sin\theta\, e^{-i\gamma} & \cos\theta \end{bmatrix}$$

The invariant is very smart!!

Generation of the Baryon Asymmetry of the Universe (BAU)

The ingredients to dynamically generate BAU from an initial state with zero B. A., were formulated by Sakharov (1967)

(i) Baryon number violation

(ii) C and CP Violation

(iii) Departure from thermal equilibrium

All these ingredients exist in the SM, but it has been established that in the SM, one cannot generate the observed BAU:

$$\boxed{\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}}$$

n_B, $n_{\bar{B}}$, n_γ number densities of baryons, antibaryons and photons at present time.

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small

$$\boxed{\frac{\mathrm{tr}\left[H_u, H_d\right]^3}{T_{ew}^{12}} \approx 10^{-20}}$$

(ii) Successful baryogenesis requires a strongly first order phase transition which would require a light Higgs mass

$$m_H \lesssim 70\,\mathrm{GeV}$$

One concludes that an explanation of the observed BAU requires New Physics beyond the SM. Leptogenesis, suggested by Fukugita and Yanagida is one the simplest and most atractive mechanisms to generate BAU:

Out of equilibrium decays of right-handed neutrinos create a lepton asymmetry which is in turn converted into a baryon asymmetry by (B+L) violating but (B-L) conserving sphaleron interactions.

The $SM\nu$, i.e. the extension of the SM consisting of adding 3 right-handed neutrinos has all the ingredients to have Leptogenesis. For an excellent review, see Sacha Davidson, E. Nardi, Y. Nir.

$$\mathcal{L}_m = -\left[\overline{\nu_L^0}\, m_D\, \nu_R^0 + \frac{1}{2}\, \nu_R^{0T} C M_R \nu_R^0 + \right.$$

$$\left. + \overline{l_L^0}\, m_l\, l_R^0\right] + h.c.$$

$$= -\left[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0}\, m_l\, l_R^0\right] + h.c.$$

with $n_L = \begin{bmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{bmatrix}$

The full neutrino mass matrix is a 6×6 matrix :

$$\mathcal{M} = \begin{bmatrix} 0 & m \\ m^T & M \end{bmatrix}$$

diagonalized by :

$$V^T \mathcal{M}^* V = \mathcal{D} \;; \quad \mathcal{D} = \text{diag.}(m_1 m_2 m_3 M_1 M_2 M_3)$$

$$V = \begin{bmatrix} K & G \\ S & T \end{bmatrix} \qquad \mathcal{D} = \begin{bmatrix} d & \\ & D \end{bmatrix}$$

unitary 6×6 matrix

One can show that :

$$S^{\dagger} \approx K^{\dagger} m M^{-1} \quad ; \quad G = m T^{*} D^{-1} \approx m D^{-1}$$

$$U_{PMNS} \leftarrow \underbrace{-K^{\dagger} m \frac{1}{M} m^{T} K^{*}}_{\text{usual seesaw formula}} = d$$

The leptonic charged current interactions are :

$$-\frac{g}{\sqrt{2}} \left(\bar{l}_{iL} \gamma_{\mu} K_{ij} \nu_{jL} + \bar{l}_{iL} \gamma_{\mu} G_{ij} N_{jL} \right) W^{\mu} + h.c.$$

Counting parameters in the leptonic sector:

Without loss of generality, one can choose a weak basis where the charged lepton mass matrix is diagonal, and also the righthanded neutrino mass matrix is diagonal. In this basis, the Yukawa coupling matrix Y_D entering in the Dirac neutrino mass matrix is an arbitrary complex matrix. 3 of the 9 phases in Y_D can be eliminated by rephasing. So altogether:

$$\underbrace{3}_{(m_\ell)_i} + \underbrace{3}_{(M_R)_i} + \underbrace{9}_{\text{real par. in } Y_D} + \underbrace{6}_{\text{phases in } Y_D} = 21 \,!$$

Lepton - Asymmetry generated through CP violating decays of the Heavy neutrinos:

$$N \to l + H$$

Unflavoured Leptogenesis:

$$A^j = \frac{\sum_i N_i^j - \bar{N}_i^j}{\sum_i N_i^j + \bar{N}_i^j} \propto \sum_{k \neq j} c_k \, \mathrm{Im}\left[(m_D^\dagger m_D)_{jk}^2\right]$$

Casas and Ibarra parametrization:

$$m_D = i \, U_\nu \sqrt{d} \; R \sqrt{D}$$

$R \to$ complex orthogonal matrix.

$$m_D^\dagger m_D = -\sqrt{D_R} \, R^\dagger \sqrt{d} \sqrt{d} \, R \sqrt{D_R}$$

Leptogenesis independent of U_ν

- In general, it is not possible to establish a connection between CP asymmetries needed for leptogenesis and CP violation detectable in neutrino oscillations. One may have leptogenesis even if U_e is real.

- The connection may be established with further theoretical assumptions.

Can one have a WB invariant which is sensitive to the CP violating phases entering in unflavoured Leptogenesis?

Yes! G.C.B., T. Morozumi, B.M. Nobre, M.N. Rebelo
Nucl. Physics B (2001)

$$I \equiv \mathrm{Im}\,\mathrm{tr}\left[h\, H\, M^* h^* M\right]$$

$$= M_1 M_2 (M_2^2 - M_1^2)\, \mathrm{Im}(h_{12}^2) + M_1 M_3 (M_3^2 - M_1^2)\, \mathrm{Im}(h_{13}^2) + M_2 M_3 (M_3^2 - M_2^2)\, \mathrm{Im}(h_{23})^2$$

$$h \equiv m_D^\dagger m_D \; ; \quad H \equiv M^\dagger M$$

Conclusions

- Neutrino Oscillations provide clear evidence for Physics beyond the SM and the discovery of $U_{e3} \neq 0$ opens up the exciting possibility of detecting leptonic Dirac-type CP violation through neutrino oscillations.
- Leptogenesis is an attractive framework to generate BAU which can occur in the framework of SMν. Difficult to test experimentally, but ...

- It is urgent to conceive "feasible experiments" which can measure physical quantities in M_ν beyond the seven quantities mentioned by Glashow et al.

  Difficult but what looks impossible today, may be possible tomorrow!

- It would be very nice if some years from now, we have a workshop with a title like:

  "The leptonic unitarity triangle fit"