

Neutrinos and Discrete Flavor Symmetries

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Neutrinos at the forefront of elementary particle physics and astrophysics

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Outline

- Data on Lepton Mixing - Indication of a Flavor Symmetry G_f ?
- Collection of Possibilities for Explaining Data
 - Non-trivial Breaking of G_f
(Lam ('07,'08), de Adelhart Toorop et al. ('11))
 - G_f and CP
(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. (in progress))
 - Sequential Breaking of G_f
(some example: *Feruglio et al. (in progress)*)
 - Comments on Model Realizations
- Conclusions

Data on Lepton Mixing

Results of latest global fits (*Gonzalez-Garcia et al. ('12)*)

best fit and 1σ error

$$\sin^2 \theta_{12} = 0.30^{+0.013}_{-0.013}$$

$$\sin^2 \theta_{23} = \begin{cases} 0.41^{+0.037}_{-0.025} \\ 0.59^{+0.021}_{-0.022} \end{cases}$$

$$\sin^2 \theta_{13} = 0.023^{+0.0023}_{-0.0023}$$

3σ range

$$0.27 \leq \sin^2 \theta_{12} \leq 0.34$$

$$0.34 \leq \sin^2 \theta_{23} \leq 0.67$$

$$0.016 \leq \sin^2 \theta_{13} \leq 0.030$$

Indication of a Flavor Symmetry G_f ?

You might answer: yes, since

$\mu\tau$ symmetry (*Fukuyama/Nishiura ('97), Mohapatra/Nussinov ('99), Lam ('01), ...*)

$$||U_{PMNS}|| = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ \frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2 \theta_{12} \text{ free}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

describes some of the data.

Indication of a Flavor Symmetry G_f ?

You might answer: yes, since

Tri-Bimaximal mixing (TB mixing) (*Harrison/Perkins/Scott ('02), Xing ('02)*)

$$||U_{PMNS}|| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

describes the data still to a certain extent well.

Indication of a Flavor Symmetry G_f ?

You might answer: yes, since

Golden Ratio mixing (*Kajiyama et al. ('07)*, *Everett/Stuart ('09)*, *Feruglio/Paris ('11)*)

$$\|U_{PMNS}\| = \begin{pmatrix} \sqrt{\frac{1}{10}(5 + \sqrt{5})} & \sqrt{\frac{2}{5 + \sqrt{5}}} & 0 \\ \frac{1}{\sqrt{5 + \sqrt{5}}} & \sqrt{\frac{1}{20}(5 + \sqrt{5})} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{5 + \sqrt{5}}} & \sqrt{\frac{1}{20}(5 + \sqrt{5})} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2 \theta_{12} \approx 0.276, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

describes the data still to a certain extent well.

Indication of a Flavor Symmetry G_f ?

You might answer: yes, since

$\Delta(96)$ Mixing (*de Adelhart Toorop et al. ('11)*)

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}(\sqrt{3} + 1) & 1 & \frac{1}{2}(\sqrt{3} - 1) \\ \frac{1}{2}(\sqrt{3} - 1) & 1 & \frac{1}{2}(\sqrt{3} + 1) \\ 1 & 1 & 1 \end{pmatrix}$$

$$\sin^2 \theta_{12} \approx 0.349, \quad \sin^2 \theta_{23} \approx \begin{cases} 0.349 \\ 0.651 \end{cases}, \quad \sin^2 \theta_{13} \approx 0.045$$

describes the data to a certain extent well.

Indication of a Flavor Symmetry G_f ?

You might answer: yes, since

$\Delta(384)$ Mixing (*de Adelhart Toorop et al. ('11)*)

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2} \sqrt{4 + \sqrt{2} + \sqrt{6}} & 1 & \frac{1}{2} \sqrt{4 - \sqrt{2} - \sqrt{6}} \\ \frac{1}{2} \sqrt{4 + \sqrt{2} - \sqrt{6}} & 1 & \frac{1}{2} \sqrt{4 - \sqrt{2} + \sqrt{6}} \\ \sqrt{1 - \frac{1}{\sqrt{2}}} & 1 & \sqrt{1 + \frac{1}{\sqrt{2}}} \end{pmatrix}$$

$$\sin^2 \theta_{12} \approx 0.337, \quad \sin^2 \theta_{23} \approx \begin{cases} 0.424 \\ 0.576 \end{cases}, \quad \sin^2 \theta_{13} \approx 0.011$$

describes the data quite well.

Indication of a Flavor Symmetry G_f ?

You could also answer: no, see e.g. [de Gouvea/Murayama \('12\)](#)

However, if you follow this line of thought, then you forget that in many models beyond the SM the symmetry G_f also helps to constrain the form of

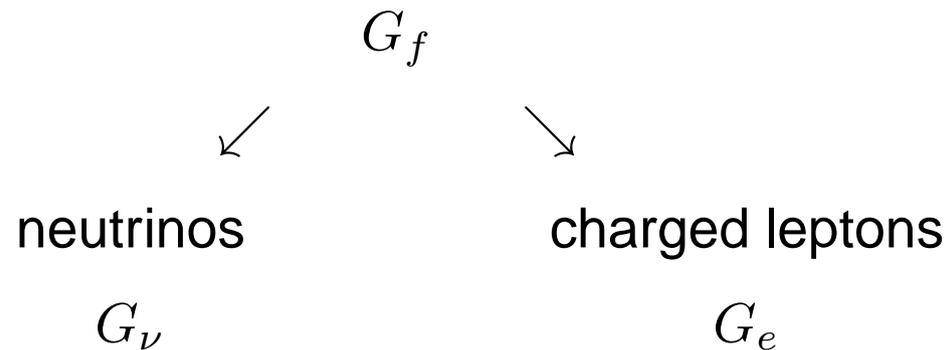
- mass matrices of additional particles
(e.g. soft terms in SUSY models)
- additional gauge interactions
(e.g. in models with gauge-Higgs unification)

in flavor space.

1st Possibility: Non-Trivial Breaking of G_f

Idea:

Derivation of the lepton mixing from how G_f is broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

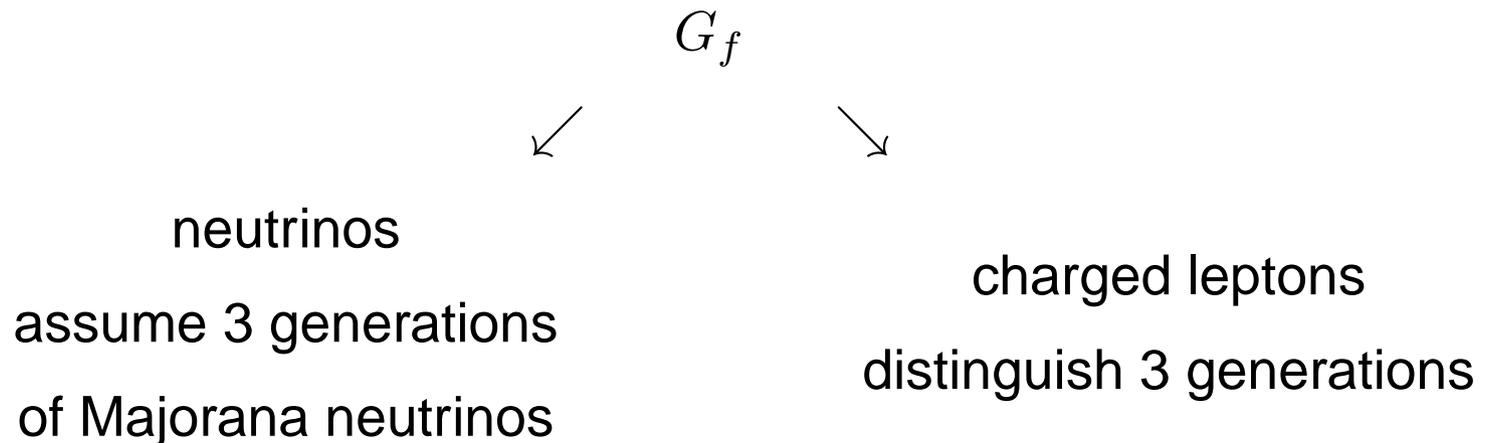


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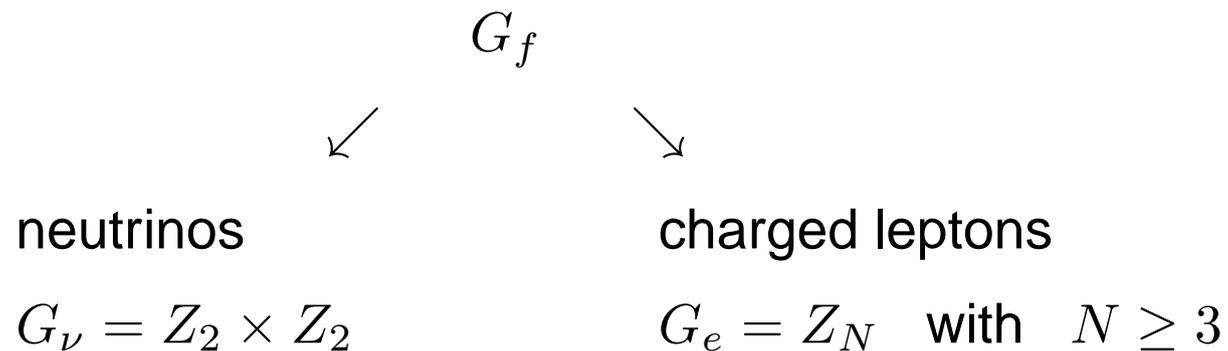


1st Possibility: Non-Trivial Breaking of G_f

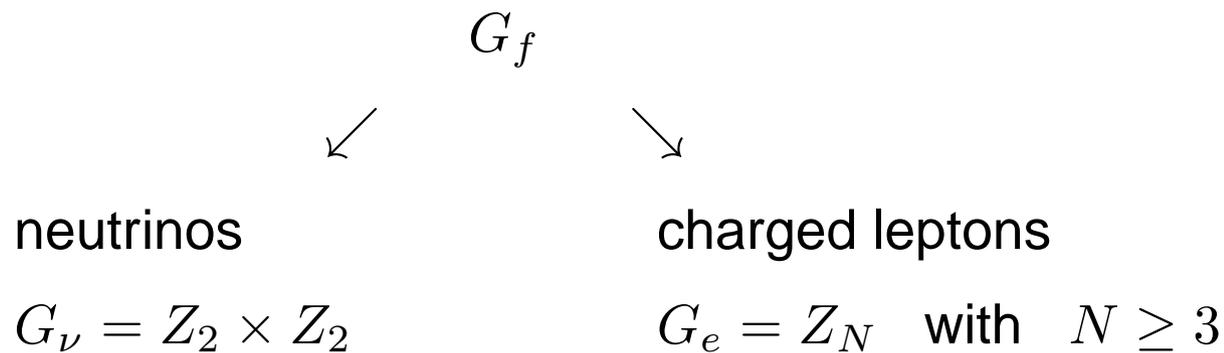
Idea:

Derivation of the lepton mixing from how G_f is broken

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1st Possibility: Non-Trivial Breaking of G_f



Further requirements

- Two/three non-trivial angles \Rightarrow irred 3-dim rep of G_f
- Fix angles through $G_\nu, G_e \Rightarrow$ 3 families transform diff. under G_ν, G_e

1st Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved

→ neutrino mass matrix m_ν fulfills

$$Z_i^T m_\nu Z_i = m_\nu \quad \text{with } i = 1, 2$$

- Charged lepton sector: Z_N , $N \geq 3$, preserved

→ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

1st Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved and generated by

$$Z_i = \Omega_\nu Z_i^{diag} \Omega_\nu^\dagger \quad \text{with } i = 1, 2$$

$$Z_i^{diag} = \text{diag}(\pm 1, \pm 1, \pm 1) \quad \text{and } \Omega_\nu \text{ unitary}$$

- Charged lepton sector: Z_N , $N \geq 3$, preserved

→ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

1st Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved

→ neutrino mass matrix m_ν fulfills

$$Z_i^{diag} [\Omega_\nu^T m_\nu \Omega_\nu] Z_i^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{with } i = 1, 2$$

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$$\Omega_\nu^T m_\nu \Omega_\nu \text{ is diagonal}$$

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- Neutrino sector: $Z_2 \times Z_2$ preserved

→ neutrino mass matrix m_ν fulfills

$$\Omega_\nu^T m_\nu \Omega_\nu \text{ is diagonal}$$

- Charged lepton sector: Z_N , $N \geq 3$, preserved and generated by

$$Q_e = \Omega_e Q_e^{diag} \Omega_e^\dagger \text{ with } \Omega_e \text{ unitary}$$

$$Q_e^{diag} = \text{diag} (\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau})$$

$$\text{and } n_e \neq n_\mu \neq n_\tau \text{ and } \omega_N = e^{2\pi i/N}$$

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→ neutrino mass matrix m_ν fulfills

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- Charged lepton sector: Z_N , $N \geq 3$, preserved

→ charged lepton mass matrix m_e fulfills

$$\Omega_e^\dagger m_e^\dagger m_e \Omega_e \text{ is diagonal}$$

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- Neutrino sector: $Z_2 \times Z_2$ preserved

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- Charged lepton sector: Z_N , $N \geq 3$, preserved

→ charged lepton mass matrix m_e fulfills

$$\Omega_e^\dagger m_e^\dagger m_e \Omega_e \text{ is diagonal}$$

- Conclusion: $\Omega_{\nu,e}$ diagonalize m_ν and $m_e^\dagger m_e$

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

1st Possibility: Non-Trivial Breaking of G_f

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

- 3 unphysical phases are removed by $\Omega_e \rightarrow \Omega_e K_e$
- Neutrino masses are made real and positive through $\Omega_\nu \rightarrow \Omega_\nu K_\nu$
- Permutations of columns of Ω_e, Ω_ν are possible: $\Omega_{e,\nu} \rightarrow \Omega_{e,\nu} P_{e,\nu}$



Predictions:

Mixing angles up to exchange of rows/columns

Dirac CP phase δ_{CP} up to π

Majorana phases undetermined

1st Possibility: An Example

TB mixing from $G_f = S_4, G_e = Z_3$

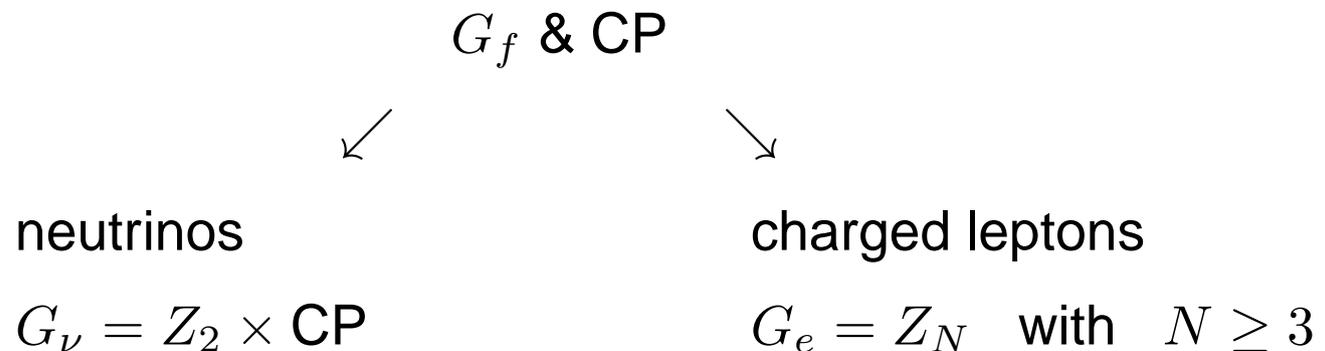
$$||U_{PMNS}|| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

2nd Possibility: G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken



An example: $\mu\tau$ reflection symmetry (*Harrison/Scott ('02,'04), Grimus/Lavoura ('03)*).

2nd Possibility: G_f and CP

In principle, procedure like in 1st case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^*$$

- Assume ϕ transforms as 3-dim rep of G_f , then

$$\phi \xrightarrow{\text{CP}} X\phi^* \xrightarrow{G_f} AX\phi^* \xrightarrow{\text{CP}} X(AX\phi^*)^* = (X^*AX)^* \phi$$

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In principle, procedure like in 1st case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^*$$

- Assume ϕ transforms as 3-dim rep of G_f , then

$$(X^*AX)^* = A' \quad \text{with} \quad A, A' \in G_f, \quad \text{but in general} \quad A \neq A'$$

2nd Possibility: G_f and CP

In principle, procedure like in 1st case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^*$$

- Assume $Z_2 \subset G_\nu$ is given by Z and is "combined" with CP

$$\phi \xrightarrow{\text{CP}} X\phi^* \xrightarrow{Z_2} ZX\phi^* \quad \text{and} \quad \phi \xrightarrow{Z_2} Z\phi \xrightarrow{\text{CP}} X(Z\phi)^*$$

2nd Possibility: G_f and CP

In principle, procedure like in 1st case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^*$$

- Assume $Z_2 \subset G_\nu$ is given by Z and is "combined" with CP

$$ZX - XZ^* = 0$$

2nd Possibility: G_f and CP

Now we just need to consider the neutrino sector: $G_\nu = Z_2 \times \text{CP}$

- Invariance conditions for m_ν

$$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^*$$

- Notice we can choose a basis such that

$$X = \Omega_\nu \Omega_\nu^T \quad \text{and} \quad Z = \Omega_\nu Z^{diag} \Omega_\nu^\dagger, \quad \Omega_\nu \text{ unitary}$$

- In this basis the conditions read

$$Z^{diag} [\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^*$$

2nd Possibility: G_f and CP

Now we just need to consider the neutrino sector: $G_\nu = Z_2 \times \text{CP}$

- In this basis the conditions read

$$Z^{diag} [\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^*$$

- Choose $Z^{diag} = \text{diag}(1, -1, 1)$
- The form of m_ν is constrained by

$$\Omega_\nu^T m_\nu \Omega_\nu = \begin{pmatrix} a & 0 & d \\ 0 & b & 0 \\ d & 0 & c \end{pmatrix} \quad \text{with } a, b, c, d \text{ real}$$

2nd Possibility: G_f and CP

Now we just need to consider the neutrino sector: $G_\nu = Z_2 \times \text{CP}$

- $\Omega_\nu^T m_\nu \Omega_\nu$ is diagonalized by

$$R(\theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \quad \text{with} \quad \tan 2\theta = \frac{2d}{c-a}$$

- Diagonal matrix $K_\nu (\pm 1, \pm i)$ renders neutrino masses positive

⇓

m_ν is diagonalized by $\Omega_\nu R(\theta) K_\nu$

2nd Possibility: G_f and CP

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \rightarrow \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- Permutations of rows and columns of U_{PMNS} possible



Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

2nd Possibility: An Example

$\mu\tau$ reflection symmetry from $G_f = S_4$, $G_e = Z_3$, one admissible X

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} \cos \theta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \sin \theta \\ -\frac{\cos \theta}{\sqrt{6}} + i \frac{\sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i \frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{6}} \\ -\frac{\cos \theta}{\sqrt{6}} - i \frac{\sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i \frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{6}} \end{pmatrix}$$

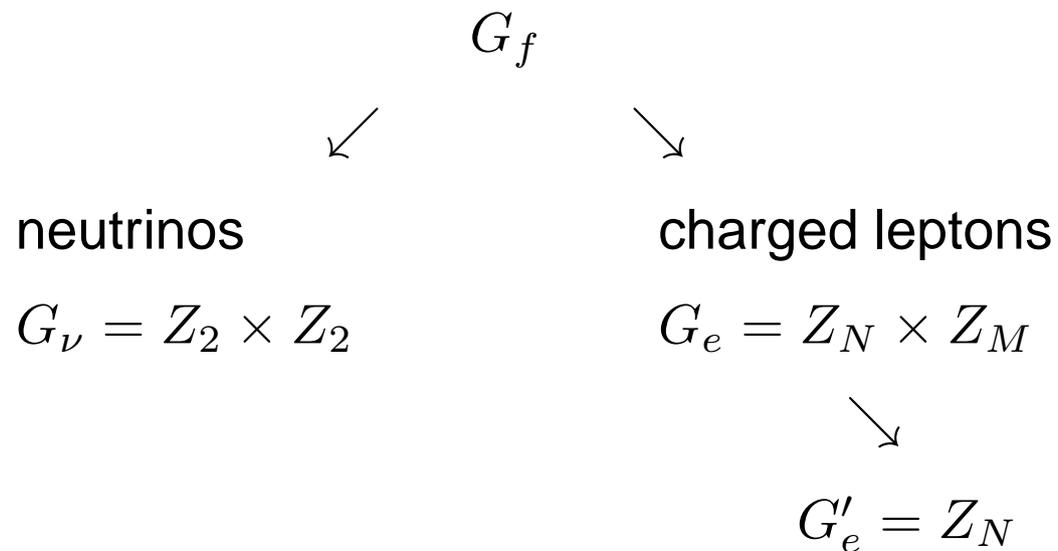
$$\sin^2 \theta_{12} = \frac{1}{2 + \cos 2\theta}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = \frac{2}{3} \sin^2 \theta$$

$$|\sin \delta_{CP}| = 1, \quad \sin \alpha = 0, \quad \sin \beta = 0$$

3rd Possibility: Sequential Breaking of G_f

Idea:

We do not break G_f to G_ν and G_e in one step, but for example we can consider this possibility



3rd Possibility: Sequential Breaking of G_f

- Consider the case $M = N = 2$
- If both symmetries are preserved, we know that the matrix $\Omega_e^\dagger m_e^\dagger m_e \Omega_e$ is diagonal
- Choose $Q_{e,1}^{diag} = \text{diag}(1, 1, -1)$ and $Q_{e,2}^{diag} = \text{diag}(1, -1, 1)$
- The second step of breaking (Z_2 given by $Q_{e,2}$ is no longer intact) allows for

$$\Omega_e^\dagger m_e^\dagger m_e \Omega_e = \begin{pmatrix} a & d & 0 \\ d^* & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

3rd Possibility: Sequential Breaking of G_f

- We thus need to apply a rotation

$$R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

in order to diagonalize $\Omega_e^\dagger m_e^\dagger m_e \Omega_e$

3rd Possibility: Sequential Breaking of G_f

- After the second step of breaking the PMNS matrix reads

$$U_{PMNS} = R(\theta)^T \Omega_e^\dagger \Omega_\nu$$

- Since it is a two-step breaking, we expect θ small
- Interesting example:
 $G_f = S_4$, $M = N = 2$; it leads to bimaximal mixing from which should be deviated by θ of order $\lambda \approx 0.22$ in order to reach agreement with data

Comments on Model Realizations

In explicit models several sources of corrections to the shown results can exist

- Higher-order corrections to $m_{\nu(e)}$ from $G_{e(\nu)}$ sector
- Corrections to the vacuum alignment from the other sector, if G_f is broken spontaneously
- Non-canonical kinetic terms
- Corrections from RG running
- ...

Conclusions

- Relation between flavor symmetry G_f , its breaking and mixing pattern
- Several ways of implementation: non-trivial breaking to $G_{e(\nu)}$, involving CP, breaking in steps
- How well these mechanisms can be realized in explicit models needs to be checked case-by-case
- In such models corrections to the above results usually arise

Thank you for your attention.