Neutrinos and Discrete Flavor Symmetries

C. Hagedorn

hagedorn@pd.infn.it

INFN, Sezione di Padova, Italy

Neutrinos at the forefront of elementary particle physics and astrophysics

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Outline

- Data on Lepton Mixing - Indication of a Flavor Symmetry G_f?
- Collection of Possibilities for Explaining Data
  - Non-trivial Breaking of G_f

    (Lam ('07,'08), de Adelhart Toorop et al. ('11))
  - G_f and CP

    (Harrison/Scott ('02,'04), Grimus/Lavoura ('03) , Feruglio et al. (in progress))
  - Sequential Breaking of G_f
    (some example: ***Feruglio et al. (in progress)***)
  - Comments on Model Realizations
- Conclusions

Data on Lepton Mixing

Results of latest global fits ***(Gonzalez-Garcia et al. ('12))***

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{12} = 0.30^{+0.013}_{-0.013}$	$0.27 \le \sin^2\theta_{12} \le 0.34$
$\sin^2\theta_{23} = \begin{cases} 0.41^{+0.037}_{-0.025} \\ 0.59^{+0.021}_{-0.022} \end{cases}$	$0.34 \le \sin^2\theta_{23} \le 0.67$
$\sin^2\theta_{13} = 0.023^{+0.0023}_{-0.0023}$	$0.016 \le \sin^2\theta_{13} \le 0.030$

Indication of a Flavor Symmetry G_f?

You might answer: yes, since

$\mu\tau$ symmetry ***(Fukuyama/Nishiura ('97), Mohapatra/Nussinov ('99), Lam ('01), ...)***

$$||U_{PMNS}|| = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} \quad \text{free} \ , \quad \sin^2\theta_{23} = \frac{1}{2} \ , \quad \sin^2\theta_{13} = 0$$

describes some of the data.

Indication of a Flavor Symmetry G_f?

You might answer: yes, since

Tri-Bimaximal mixing (TB mixing) ***(Harrison/Perkins/Scott ('02), Xing ('02))***

$$||U_{PMNS}|| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}\,, \quad \sin^2\theta_{23} = \frac{1}{2}\,, \quad \sin^2\theta_{13} = 0$$

describes the data still to a certain extent well.

Indication of a Flavor Symmetry G_f?

You might answer: yes, since

Golden Ratio mixing ***(Kajiyama et al. ('07), Everett/Stuart ('09), Feruglio/Paris ('11))***

$$||U_{PMNS}|| = \begin{pmatrix} \sqrt{\frac{1}{10}(5+\sqrt{5})} & \sqrt{\frac{2}{5+\sqrt{5}}} & 0 \\ \frac{1}{\sqrt{5+\sqrt{5}}} & \sqrt{\frac{1}{20}(5+\sqrt{5})} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{5+\sqrt{5}}} & \sqrt{\frac{1}{20}(5+\sqrt{5})} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} \approx 0.276 \; , \;\; \sin^2\theta_{23} = \frac{1}{2} \; , \;\;\; \sin^2\theta_{13} = 0$$

describes the data still to a certain extent well.

Indication of a Flavor Symmetry G_f?

You might answer: yes, since

$\Delta(96)$ Mixing ***(de Adelhart Toorop et al. ('11))***

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}(\sqrt{3}+1) & 1 & \frac{1}{2}(\sqrt{3}-1) \\ \frac{1}{2}(\sqrt{3}-1) & 1 & \frac{1}{2}(\sqrt{3}+1) \\ 1 & 1 & 1 \end{pmatrix}$$

$$\sin^2\theta_{12} \approx 0.349 \; , \;\; \sin^2\theta_{23} \approx \left\{ \begin{array}{c} 0.349 \\ 0.651 \end{array} \right. \; , \;\; \sin^2\theta_{13} \approx 0.045$$

describes the data to a certain extent well.

Indication of a Flavor Symmetry G_f?

You might answer: yes, since

$\Delta(384)$ Mixing ***(de Adelhart Toorop et al. ('11))***

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\sqrt{4+\sqrt{2}+\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}-\sqrt{6}} \\ \frac{1}{2}\sqrt{4+\sqrt{2}-\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}+\sqrt{6}} \\ \sqrt{1-\frac{1}{\sqrt{2}}} & 1 & \sqrt{1+\frac{1}{\sqrt{2}}} \end{pmatrix}$$

$$\sin^2\theta_{12} \approx 0.337 \; , \;\; \sin^2\theta_{23} \approx \begin{cases} 0.424 \\ 0.576 \end{cases} \; , \;\; \sin^2\theta_{13} \approx 0.011$$

describes the data quite well.

Indication of a Flavor Symmetry G_f?

You could also answer: no, see e.g. ***de Gouvea/Murayama ('12)***
However, if you follow this line of thought, then you forget that in many models beyond the SM the symmetry G_f also helps to constrain the form of

- mass matrices of additional particles
  (e.g. soft terms in SUSY models)
- additional gauge interactions
  (e.g. in models with gauge-Higgs unification)

in flavor space.

1^{st} Possibility: Non-Trivial Breaking of G_f

<u>Idea</u>:

Derivation of the lepton mixing from how G_f is broken
Interpretation as mismatch of embedding of different sub-groups G_ν and G_e into G_f

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos — charged leptons

G_ν — G_e

1^{st} Possibility: Non-Trivial Breaking of G_f

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Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos
assume 3 generations
of Majorana neutrinos

charged leptons
distinguish 3 generations

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Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

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neutrinos — charged leptons

$G_\nu = Z_2 \times Z_2$ — $G_e = Z_N$ with $N \geq 3$

1^{st} Possibility: Non-Trivial Breaking of G_f

$$G_f$$

$$\swarrow \qquad\qquad\qquad \searrow$$

neutrinos — $G_\nu = Z_2 \times Z_2$

charged leptons — $G_e = Z_N$ with $N \geq 3$

Further requirements

- Two/three non-trivial angles $\Rightarrow$ irred 3-dim rep of G_f
- Fix angles through G_ν, G_e $\Rightarrow$ 3 families transform diff. under G_ν, G_e

1^{st} Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $$Z_i^T m_\nu Z_i = m_\nu \quad \text{with} \quad i = 1, 2$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

1^{st} Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved and generated by

$$Z_i = \Omega_\nu Z_i^{diag} \Omega_\nu^\dagger \quad \text{with} \quad i = 1, 2$$

$$Z_i^{diag} = \text{diag}\,(\pm 1, \pm 1, \pm 1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

1^{st} Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z_i^{diag} \left[\Omega_\nu^T m_\nu \Omega_\nu\right] Z_i^{diag} = \left[\Omega_\nu^T m_\nu \Omega_\nu\right] \quad \text{with} \quad i = 1, 2$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

1^{st} Possibility: Non-Trivial Breaking of G_f

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  $$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

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- Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

  $$Q_e = \Omega_e Q_e^{diag} \Omega_e^\dagger \quad \text{with} \quad \Omega_e \quad \text{unitary}$$
  $$Q_e^{diag} = \text{diag}\left(\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau}\right)$$
  $$\text{and} \quad n_e \neq n_\mu \neq n_\tau \quad \text{and} \quad \omega_N = e^{2\pi i/N}$$

1^{st} Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $$\Omega_e^\dagger m_e^\dagger m_e \Omega_e \quad \text{is diagonal}$$

$1^{\rm st}$ Possibility: Non-Trivial Breaking of G_f

- Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $\Omega_\nu^T m_\nu \Omega_\nu$ is diagonal

- Charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $\Omega_e^\dagger m_e^\dagger m_e \Omega_e$ is diagonal

- Conclusion: $\Omega_{\nu,e}$ diagonalize m_ν and $m_e^\dagger m_e$

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

1^{st} Possibility: Non-Trivial Breaking of G_f

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

- 3 unphysical phases are removed by $\Omega_e \rightarrow \Omega_e K_e$
- Neutrino masses are made real and positive through $\Omega_\nu \rightarrow \Omega_\nu K_\nu$
- Permutations of columns of Ω_e, Ω_ν are possible: $\Omega_{e,\nu} \rightarrow \Omega_{e,\nu} P_{e,\nu}$

$$\Downarrow$$

Predictions:

Mixing angles up to exchange of rows/columns

Dirac CP phase δ_{CP} up to π

Majorana phases undetermined

1^{st} Possibility: An Example

TB mixing from $G_f = S_4$, $G_e = Z_3$

$$||U_{PMNS}|| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}\ , \ \ \sin^2\theta_{23} = \frac{1}{2}\ , \quad \sin^2\theta_{13} = 0$$

2^{nd} Possibility: G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken

$$G_f \text{ \& CP}$$

$$\swarrow \qquad \qquad \searrow$$

neutrinos — $G_\nu = Z_2 \times$ CP

charged leptons — $G_e = Z_N$ with $N \geq 3$

An example: $\mu\tau$ reflection symmetry *(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))*.

2nd Possibility: G_f and CP

In principle, procedure like in 1^{st} case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- Assume ϕ transforms as 3-dim rep of G_f, then

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{G_f} AX\phi^\star \xrightarrow{\text{CP}} X\left(AX\phi^\star\right)^\star = \left(X^\star AX\right)^\star \phi$$

2^{nd} Possibility: G_f and CP

In principle, procedure like in 1^{st} case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- Assume ϕ transforms as 3-dim rep of G_f, then

$$(X^\star A X)^\star = A' \quad \text{with} \quad A, A' \in G_f \ , \quad \text{but in general} \quad A \neq A'$$

2^{nd} Possibility: G_f and CP

In principle, procedure like in 1^{st} case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- Assume $Z_2 \subset G_\nu$ is given by Z and is "combined" with CP

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{Z_2} ZX\phi^\star \quad \text{and} \quad \phi \xrightarrow{Z_2} Z\phi \xrightarrow{\text{CP}} X\left(Z\phi\right)^\star$$

$2^{\rm nd}$ Possibility: G_f and CP

In principle, procedure like in $1^{\rm st}$ case, but some consistency conditions have to be fulfilled:

- Definition of generalized CP transformation (X being unitary and symmetric)

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- Assume $Z_2 \subset G_\nu$ is given by Z and is "combined" with CP

$$ZX - XZ^\star = 0$$

$2^{\rm nd}$ Possibility: G_f and CP

Now we just need to consider the neutrino sector: $G_\nu = Z_2 \times$ CP

- Invariance conditions for m_ν

$$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^\star$$

- Notice we can choose a basis such that

$$X = \Omega_\nu \Omega_\nu^T \quad \text{and} \quad Z = \Omega_\nu Z^{diag} \Omega_\nu^\dagger \quad , \quad \Omega_\nu \text{ unitary}$$

- In this basis the conditions read

$$Z^{diag}[\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

2^{nd} Possibility: G_f and CP

Now we just need to consider the neutrino sector: $G_\nu = Z_2 \times$ CP

- In this basis the conditions read

$$Z^{diag}[\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

- Choose $\quad Z^{diag} = \text{diag}\,(1, -1, 1)$
- The form of m_ν is constrained by

$$\Omega_\nu^T m_\nu \Omega_\nu = \begin{pmatrix} a & 0 & d \\ 0 & b & 0 \\ d & 0 & c \end{pmatrix} \quad \text{with} \quad a, b, c, d \text{ real}$$

2^{nd} Possibility: G_f and CP

Now we just need to consider the neutrino sector: $G_\nu = Z_2 \times$ CP

- $\Omega_\nu^T m_\nu \Omega_\nu$ is diagonalized by

$$R(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad \text{with} \quad \tan 2\theta = \frac{2d}{c-a}$$

- Diagonal matrix K_ν $(\pm 1, \pm i)$ renders neutrino masses positive

$$\Downarrow$$

m_ν is diagonalized by $\Omega_\nu R(\theta) K_\nu$

2^{nd} Possibility: G_f and CP

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- Permutations of rows and columns of U_{PMNS} possible

$$\Downarrow$$

Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

2^{nd} Possibility: An Example

$\mu\tau$ reflection symmetry from $G_f = S_4$, $G_e = Z_3$, one admissible X

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\theta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + i\,\frac{\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i\,\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}} \\ -\frac{\cos\theta}{\sqrt{6}} - i\,\frac{\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i\,\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}\ , \quad \sin^2\theta_{23} = \frac{1}{2}\ , \quad \sin^2\theta_{13} = \frac{2}{3}\sin^2\theta$$

$$|\sin\delta_{CP}| = 1\ , \quad \sin\alpha = 0\ , \quad \sin\beta = 0$$

3^{rd} Possibility: Sequential Breaking of G_f

Idea:

We do not break G_f to G_ν and G_e in one step,
but for example we can consider this possibility

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos — charged leptons

$$G_\nu = Z_2 \times Z_2 \qquad\qquad G_e = Z_N \times Z_M$$

$$\searrow$$

$$G'_e = Z_N$$

3^{rd} Possibility: Sequential Breaking of G_f

- Consider the case $M = N = 2$
- If both symmetries are preserved, we know that the matrix $\Omega_e^\dagger m_e^\dagger m_e \Omega_e$ is diagonal
- Choose $Q_{e,1}^{diag} =$ diag $(1, 1, -1)$ and $Q_{e,2}^{diag} =$ diag $(1, -1, 1)$
- The second step of breaking (Z_2 given by $Q_{e,2}$ is no longer intact) allows for

$$\Omega_e^\dagger m_e^\dagger m_e \Omega_e = \begin{pmatrix} a & d & 0 \\ d^\star & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

3^{rd} Possibility: Sequential Breaking of G_f

- We thus need to apply a rotation

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

in order to diagonalize $\Omega_e^\dagger m_e^\dagger m_e \Omega_e$

$3^{\rm rd}$ Possibility: Sequential Breaking of $\boldsymbol{G_f}$

- After the second step of breaking the PMNS matrix reads

$$U_{PMNS} = R(\theta)^T \Omega_e^\dagger \Omega_\nu$$

- Since it is a two-step breaking, we expect θ small
- Interesting example:
  $G_f = S_4$, $M = N = 2$; it leads to bimaximal mixing from which should be deviated by θ of order $\lambda \approx 0.22$ in order to reach agreement with data

Comments on Model Realizations

In explicit models several sources of corrections to the shown results can exist

- Higher-order corrections to $m_{\nu(e)}$ from $G_{e(\nu)}$ sector
- Corrections to the vacuum alignment from the other sector, if G_f is broken spontaneously
- Non-canonical kinetic terms
- Corrections from RG running
- ...

Conclusions

- Relation between flavor symmetry G_f, its breaking and mixing pattern
- Several ways of implementation: non-trivial breaking to $G_{e(\nu)}$, involving CP, breaking in steps
- How well these mechanisms can be realized in explicit models needs to be checked case-by-case
- In such models corrections to the above results usually arise

Thank you for your attention.