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Flavor stability analysis of self-induced SN neutrino oscillations

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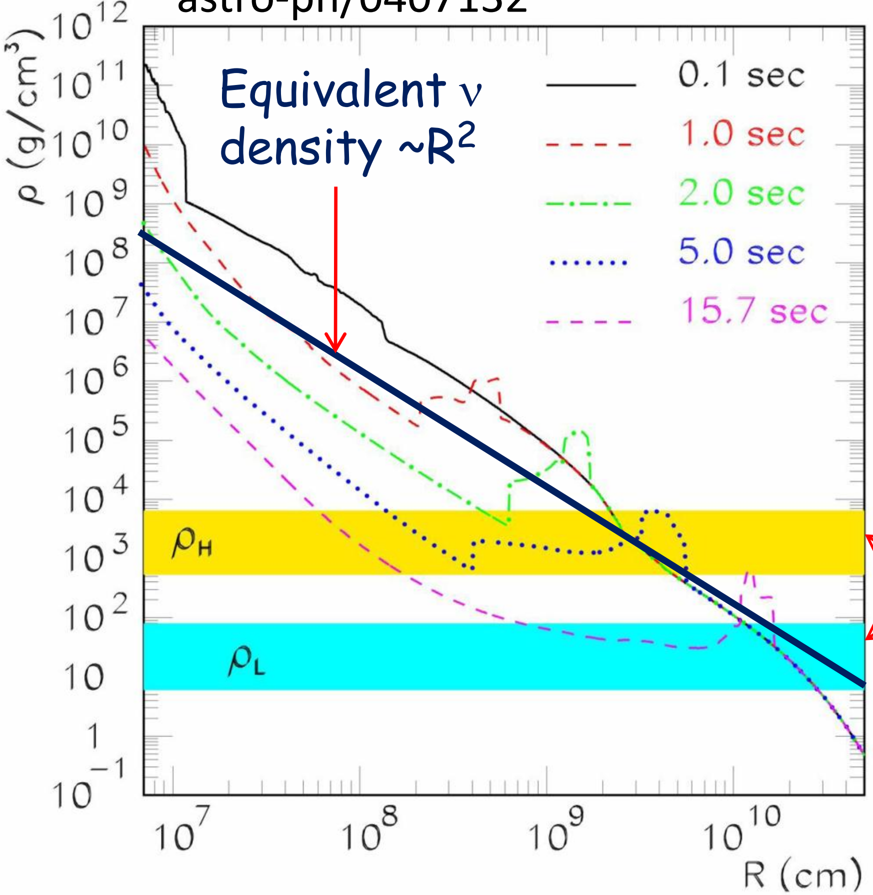
OUTLINE

- Self-induced SN ν conversions
- Linearized flavor stability analysis
- Two applications:
  - -Matter induced multi-angle suppression
  - - Instability with flavor dependent angular distributions
- Conclusions

[Based on works in collaboration with: Sovan Chakraborty, Ninetta Saviano, Pasquale Serpico]

SNAP-SHOTS OF SN DENSITY PROFILES

astro-ph/0407132



- Matter bkg potential

$$\lambda = \sqrt{2} G_F N_e \quad \sim R^{-3}$$

- ν–ν interaction

$$\mu = \sqrt{2} G_F n_\nu \quad \sim R^{-2}$$

- Vacuum oscillation frequencies

$$\omega = \frac{\Delta m^2}{2E}$$

When $\mu \gg \lambda$, SN ν oscillations dominated by ν-ν interactions

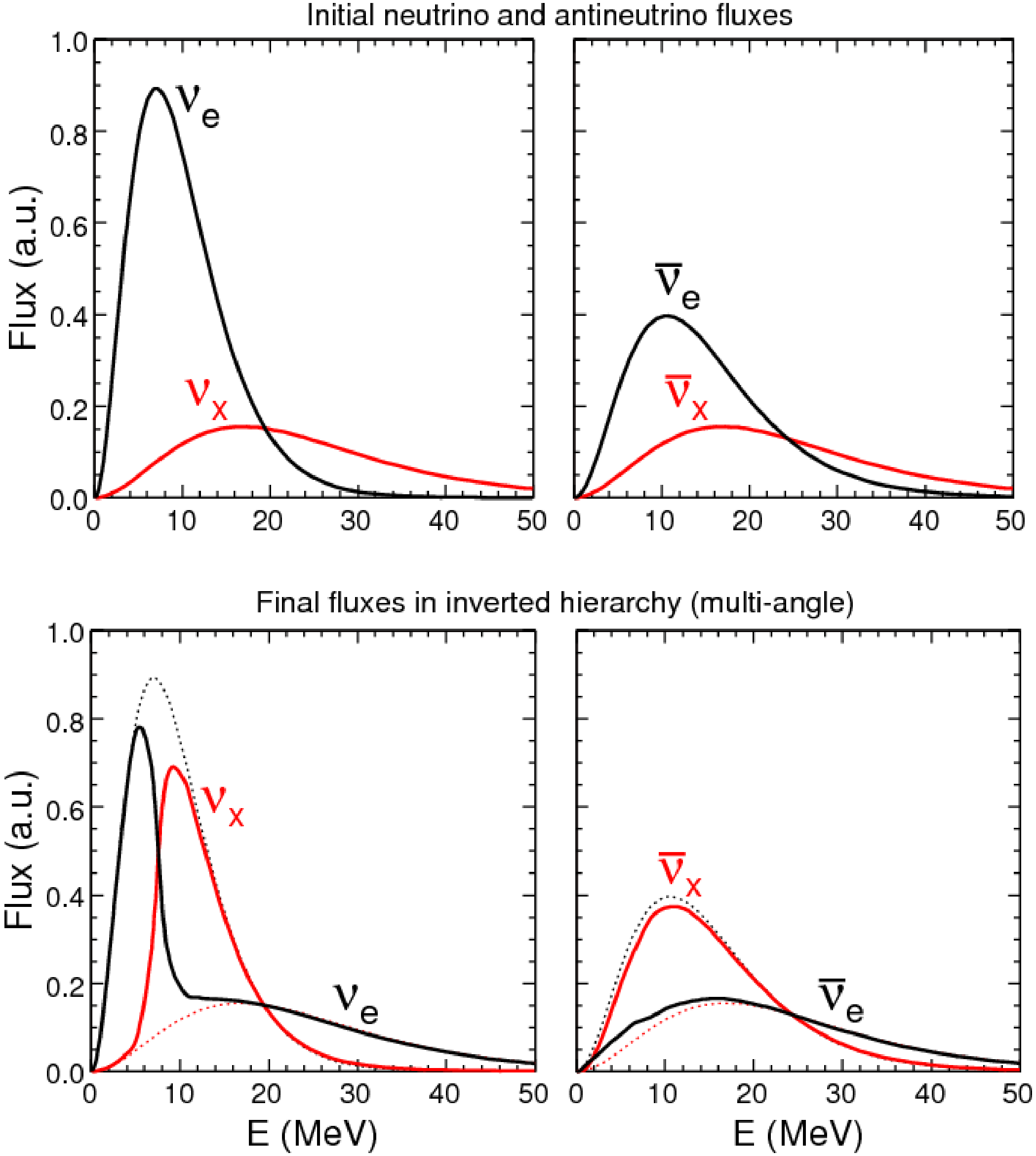
Collective flavor transitions at low-radii [O (10^2 – 10^3 km)]

Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

[see Duan et al, arXiv:1001.2799 for a review]

SELF-INDUCED SPECTRAL SPLITS

[e.g.,Fogli, Lisi, Marrone, A.M., 0707.1998]



Initial fluxes at the neutrinosphere

After collective transformations

DENSITY MATRIX FOR THE NEUTRINO ENSEMBLE

Diagonal elements related to flavor content

$$\rho_{\alpha\alpha} = \frac{F_{\nu_\alpha}(E,r)}{F(E,r)}$$

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} & \rho_{e\tau} \\ \rho^{*}_{e\mu} & \rho_{\mu\mu} & \rho_{\mu\tau} \\ \rho_{e\tau} & \rho^{*}_{\mu\tau} & \rho_{\tau\tau} \end{pmatrix}$$

Off-diagonal elements responsible for flavor conversions

The EOMs for the time evolution in a homogeneous medium are the Liouville equations (e.g. Early Universe) → *Saviano talk tomorrow*

$$i\partial_t \rho_{\mathrm{p}} = [H_{\mathrm{p}}, \rho_{\mathrm{p}}]$$

MULTI-ANGLE (M.A.) EOMs FOR SN NEUTRINOS

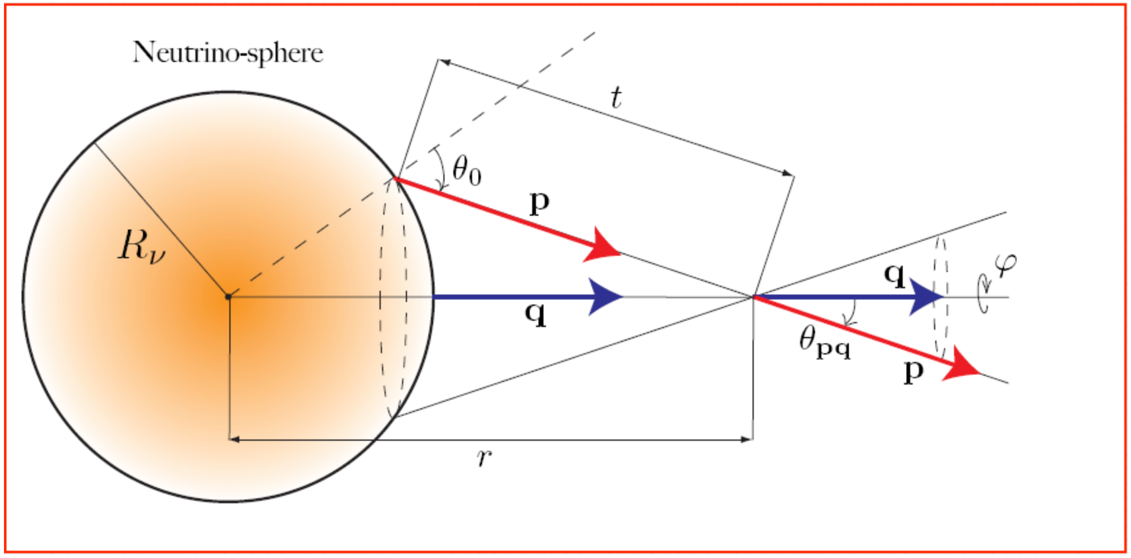
Evolution in space for ν's streaming from a SN core in quasi-stationary situation

$$i\, \vec{\mathrm{v}}_{\mathrm{p}} \cdot \vec{\nabla}_x \rho_{p,x} = \left[H(\omega, \lambda, \rho_{p',x}), \rho_{p,x} \right]$$

Liouville operator for free streaming ν

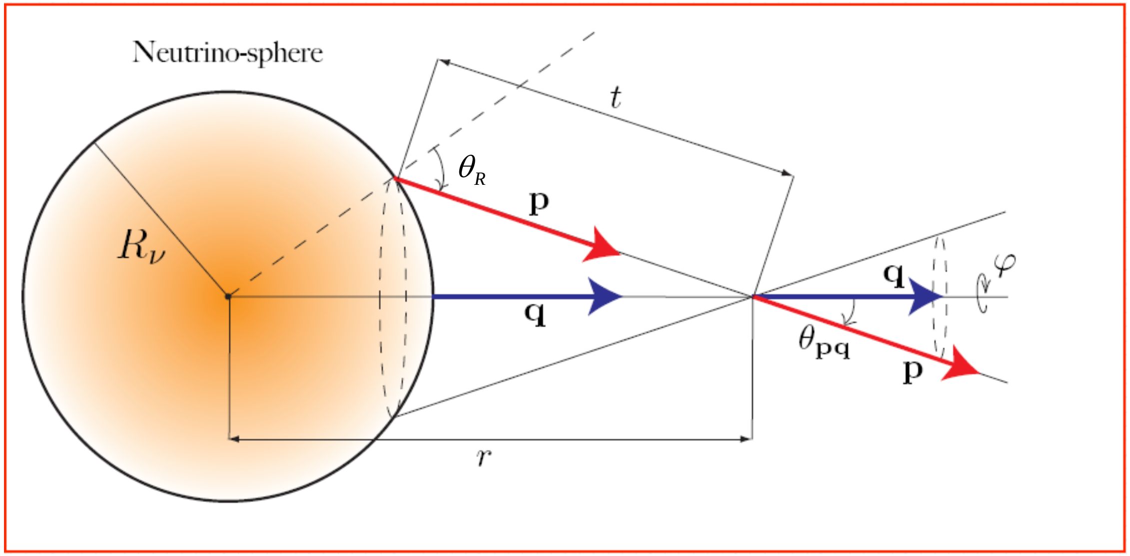
MULTI-ANGLE ν–ν HAMILTONIAN

$$H_{\nu\nu} = \sqrt{2} G_F \int d\vec{q} \left(1 - \vec{\mathrm{v}}_{\mathrm{p}} \cdot \vec{\mathrm{v}}_q\right) \left(\rho_{q,x} - \bar{\rho}_{q,x}\right)$$



BULB MODEL

[see, e.g., Duan et al., astro-ph/0606616] ⟶ First large-scale multi-angle simulations

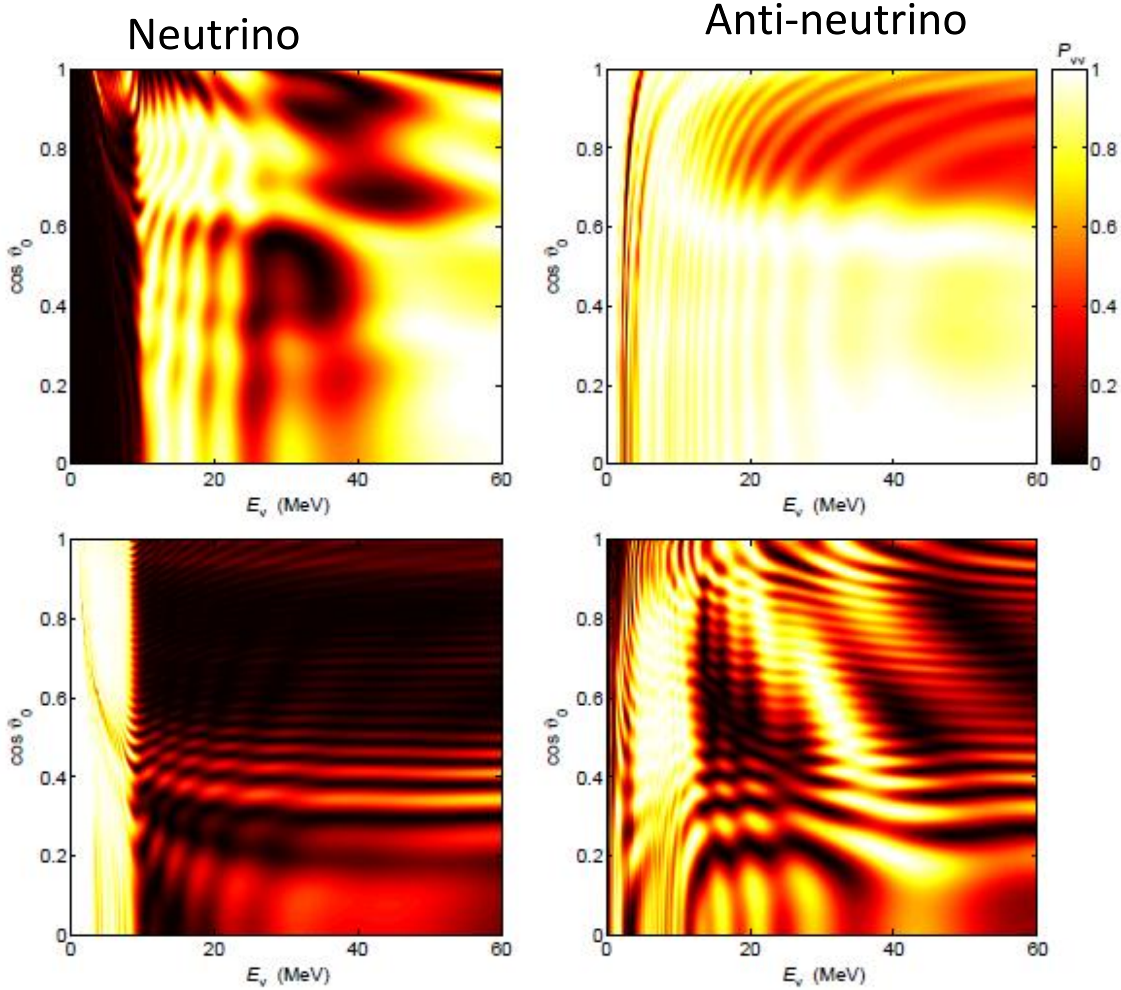


- Neutrinos are emitted uniformly and (half)-isotropically from the surface of a sphere (ν-sphere), like in a blackbody.
- Energy-independent half-isotropic angular distributions for all flavors
- Physical conditions depend only on the the distance r from the center of the star (azimuthal symmetry)
- Project evolution along radial direction (ODE problem) $\vec{\mathrm{v}}_p \cdot \vec{\nabla}_x \rightarrow \mathrm{v}_r d_r$

MULTI-ANGLE LARGE SCALE SIMULATIONS

First multi-angle simulations in 2006 by Duan, Fuller, Qian (2006). Major breakthrough!

Survival probability of ν_e vs E and emission angle cos θ



Normal hierarchy

Inverted hierarchy

Significant angular dependence on the Pee

Convergence required > 10^3 angular bins ⟶ Large scale numerical simulations

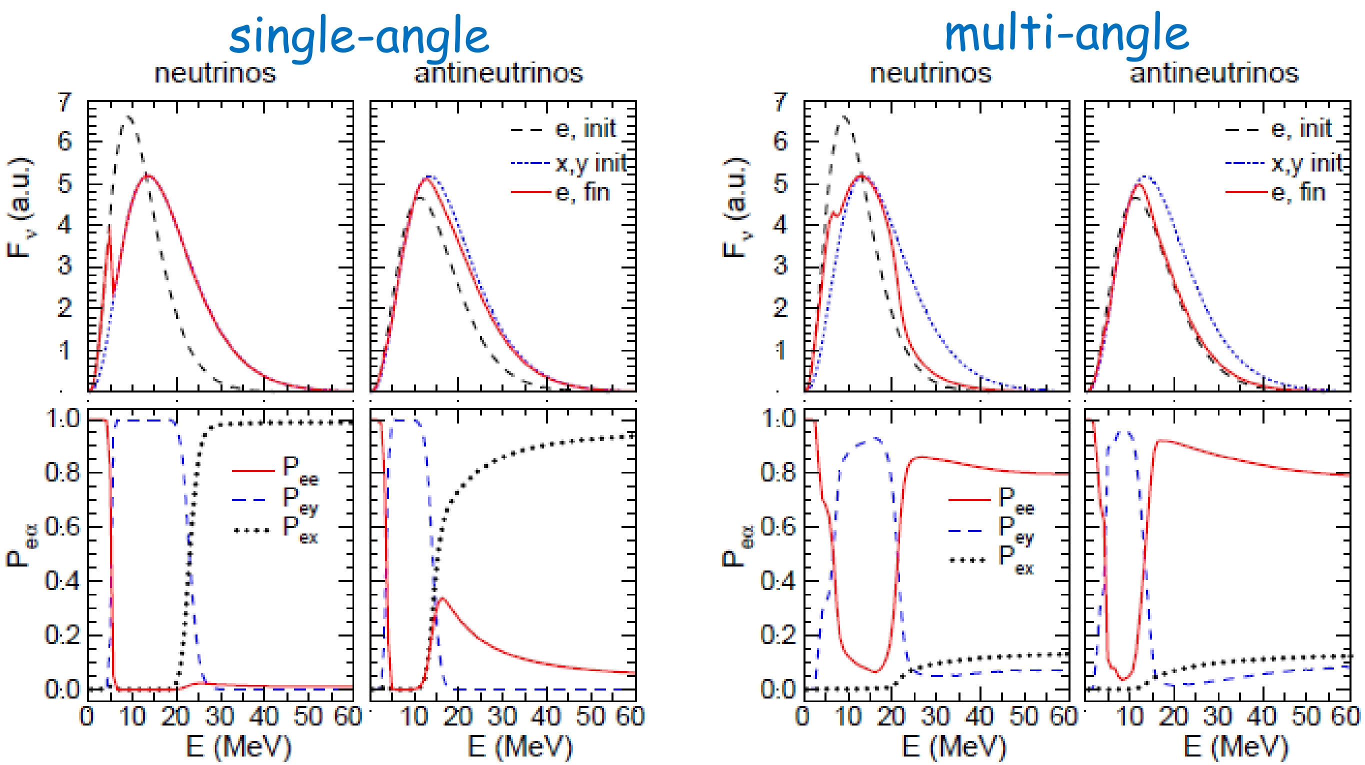
MULTI-ANGLE SIMULATIONS BY DIFFERENT GROUPS

- *Duan, Fuller, Carlson & Qian, astro-ph/0606616, 0608050*
- *Fogli, Lisi, Marrone & A.M., 0707.1998, Fogli, Lisi, Marrone, A.M & Tamborra, 0808.0807;*
- *Esteban-Pretel, Pastor, Tomas, Raffelt & Sigl, 0706.2498*
- *Duan & Friedland, 1006.2359*
- *A.M. & Tomas, 1012.1339*
- *Cherry, Fuller, Carlson, Duan, Qian, 1006.2175*

APPROXIMATE SOLUTIONS

- Single-angle approximation

Introduce an average of the multi-angle factor.
Advantages: More amenable numerically. It allows for analytical interpretations.
Disadvantages: Predictions on the SN ν flavor evolution based on single-angle approximations have too be taken *cum grano salis.* Found cases with strong differences btw SA and MA evolution.



[A.M. & Tomas, 1012.1339]

LINEARIZED FLAVOR STABILITY ANALYSIS

[*Sawyer,0803.4319; Banerjee, Dighe & Raffelt, 1107.2308*]

- Self-induced flavor conversions are associated to an instability in the flavor space

- Instability required to get started (exponential growth of the off-diagonal density matrix part)

- The onset of the conversions can be found through a stability analysis of the linearized EOMs.

- Stability analysis can tell us <u>only</u> if the system is stable or unstable.

- In the unstable case, numerical simulations are mandatory.

OSCILLATIONS VIA STABILITY ANALYSIS

[*Banerjee, Dighe & Raffelt, 1107.2308*]

- $$i\partial_r \Phi_{E,u} = \left[H_{E,u}, \Phi_{E,u}\right]$$ EOMs, u = sin²(emission angle),

- $$H_{E,u} = \frac{1}{v_u}\Big(\underbrace{\frac{M^2}{2E}}_{\text{vacuum}} + \underbrace{\sqrt{2}G_F N_e}_{\text{matter}}\Big) \underbrace{+ \sqrt{2}G_F \int dE' \int du' \left(\frac{1 - v_u v_{u'}}{v_u v_{u'}}\right)\Phi_{E',u'}}_{\nu-\nu}$$ Hamiltonian; v_u radial velocity

- $$\Phi_{\omega,u} = \frac{Tr\,\Phi_{\omega,u}}{2} + \frac{g_{\omega,u}}{2}\begin{pmatrix} s_{\omega,u} & S_{\omega,u} \\ S^*_{\omega,u} & s_{\omega,u} \end{pmatrix}$$ Neutrino flux matrices
  $\omega = \Delta m^2/2E$
  $g_{\omega,u}$ ν spectra

- Small amplitude limit: $s_{\omega,u} \sim 1$, $S_{\omega,u} \ll 1$. Linearize in small off-diagonal flux terms $S_{\omega,u}$

LINEARIZED EOMs

$$i\partial_r S_{\omega,u} = [\omega + u(\lambda + \varepsilon\mu)]S_{\omega,u} - \mu\int du' d\omega' (u+u') g_{\omega',u'} S_{\omega',u'}$$

- $\varepsilon = \int du\, d\omega\, g_{\omega,u}$ "Asymmetry" of the neutrino spectrum
- $\mu = \frac{\sqrt{2} G_F n_{\bar{\nu}_e}(R)}{4\pi r^2} \frac{R^2}{2r^2}$ $\nu-\nu$ interaction strength
- $\lambda = \sqrt{2} G_F n_e \frac{R^2}{2r^2}$ ordinary matter term

- Look for solutions in the form $S_{\omega,u} = Q_{\omega,u} e^{-i\Omega r}$
- Eigenvalue equation for $Q_{\omega,u}$

$$\left[\omega + u(\lambda + \varepsilon\mu) - \Omega\right] Q_{\omega,u} = \int du' d\omega' (u + u') g_{\omega',u'} Q_{\omega',u'}$$

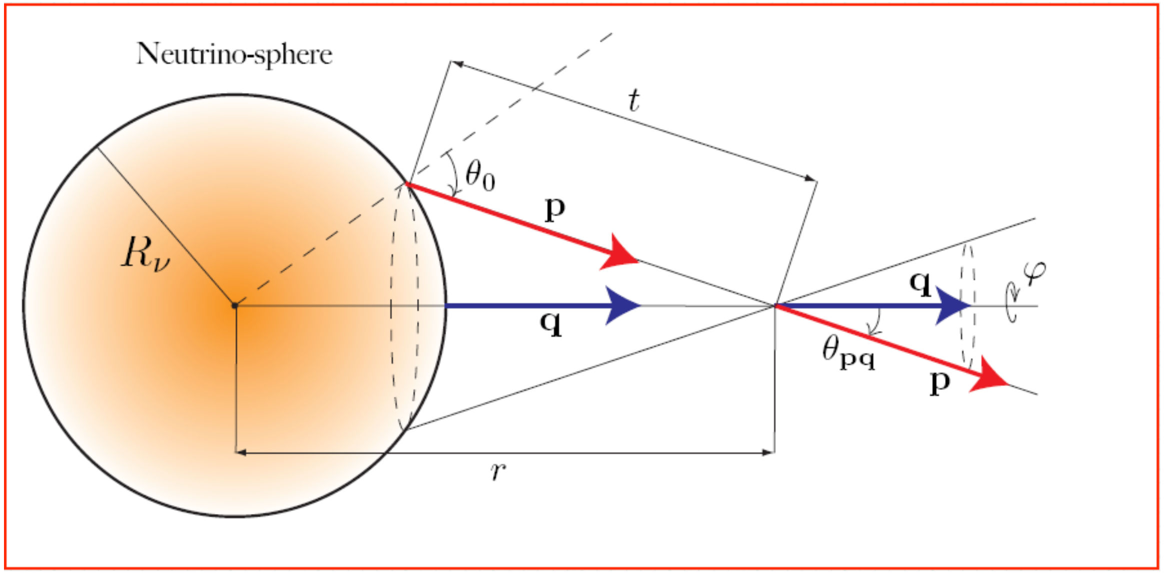
- A complex solution $\Omega = \gamma + i\kappa$, with k > 0, indicates an exponentially increasing $S_{\omega,u}$ $\rightarrow$ an instability

- The question of the stability is reduced to an eigenvalue problem

MATTER INDUCED MULTI-ANGLE EFFECTS

[*Esteban-Pretel, A.M., Pastor, Tomas, Raffelt, Serpico & Sigl, arxiv: 0807.0659*]

- Spherical stream



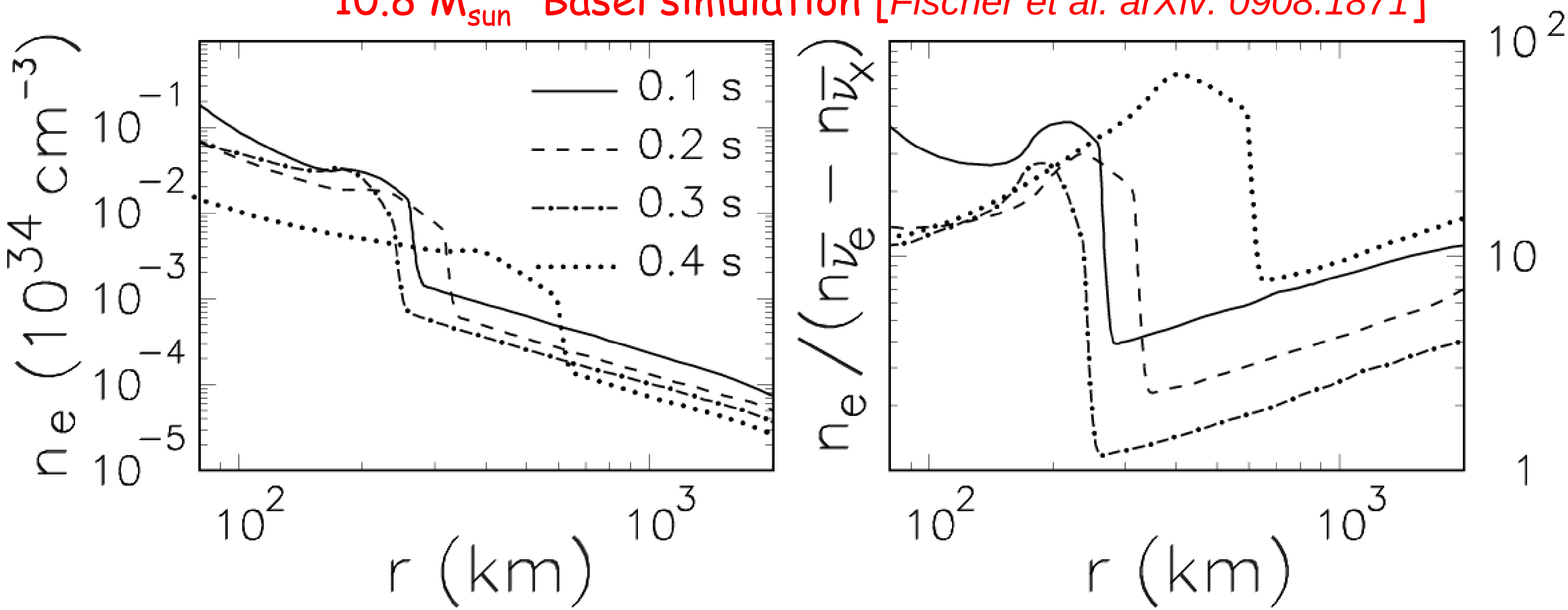
- Neutrinos emitted from a spherical source acquire different phases at a given radius r, having travelled on different trajectories.
- Matter effect is not the same for all the modes.
- It would introduce trajectory-dependent multi-angle effects.

Matter effect can suppress collective conversion unless $N_\nu \gtrsim N_e$

MATTER EFFECTS DURING THE ACCRETION PHASE

[*Chakraborthy, Fischer, A.M., Saviano & Tomas, 1104.4031 (PRL), 1105.1130*]

10.8 M_{sun} Basel simulation [*Fischer et al. arXiv: 0908.1871*]

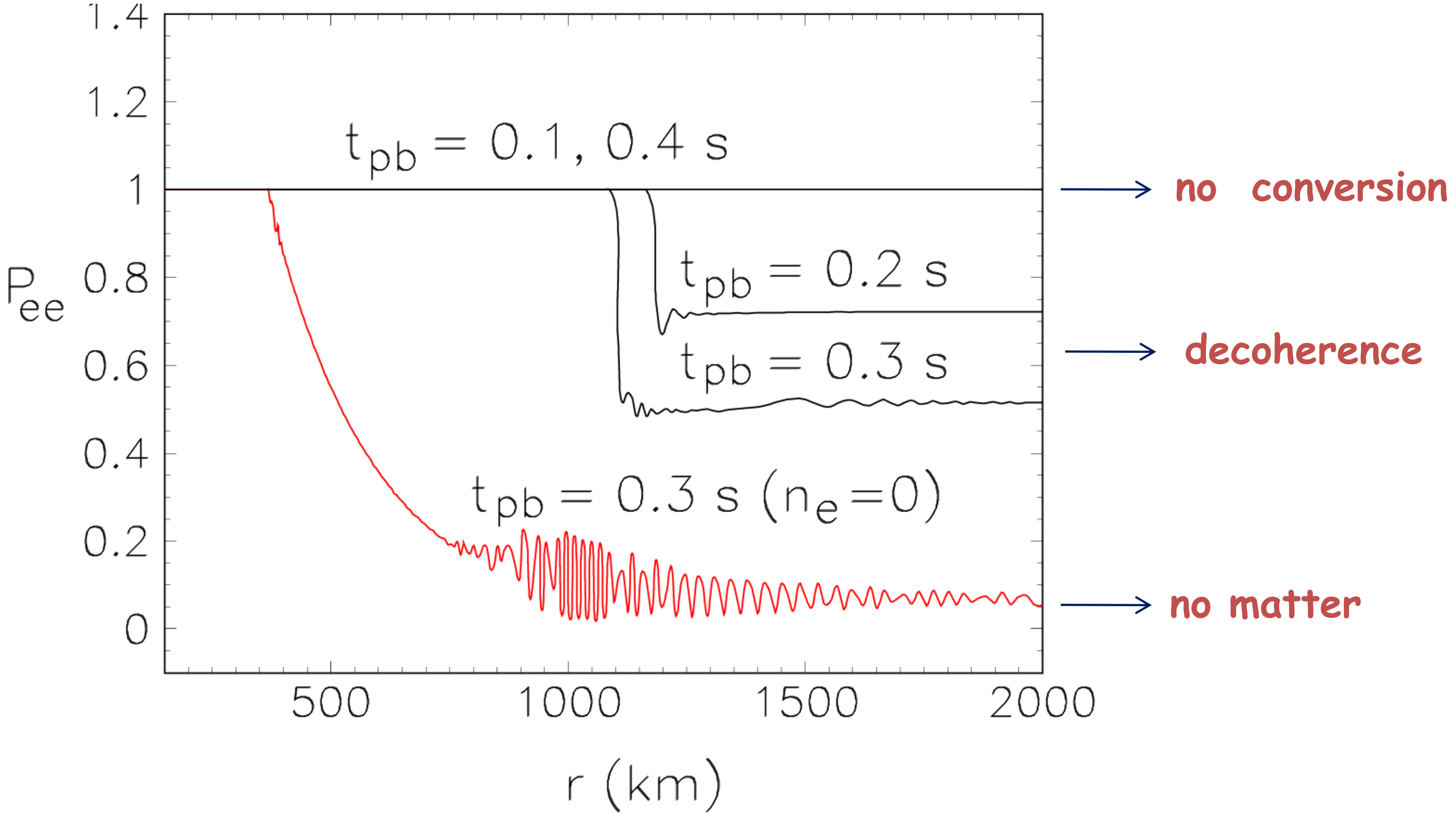


$n_e >> n_\nu$	**No flavor conversion**
$n_e \geq n_\nu$	**Multi-angle decoherence**
$n_e << n_\nu$	**Collective oscillations**

MATTER SUPPRESSION RELEVANT DURING THE ACCRETION PHASE !!

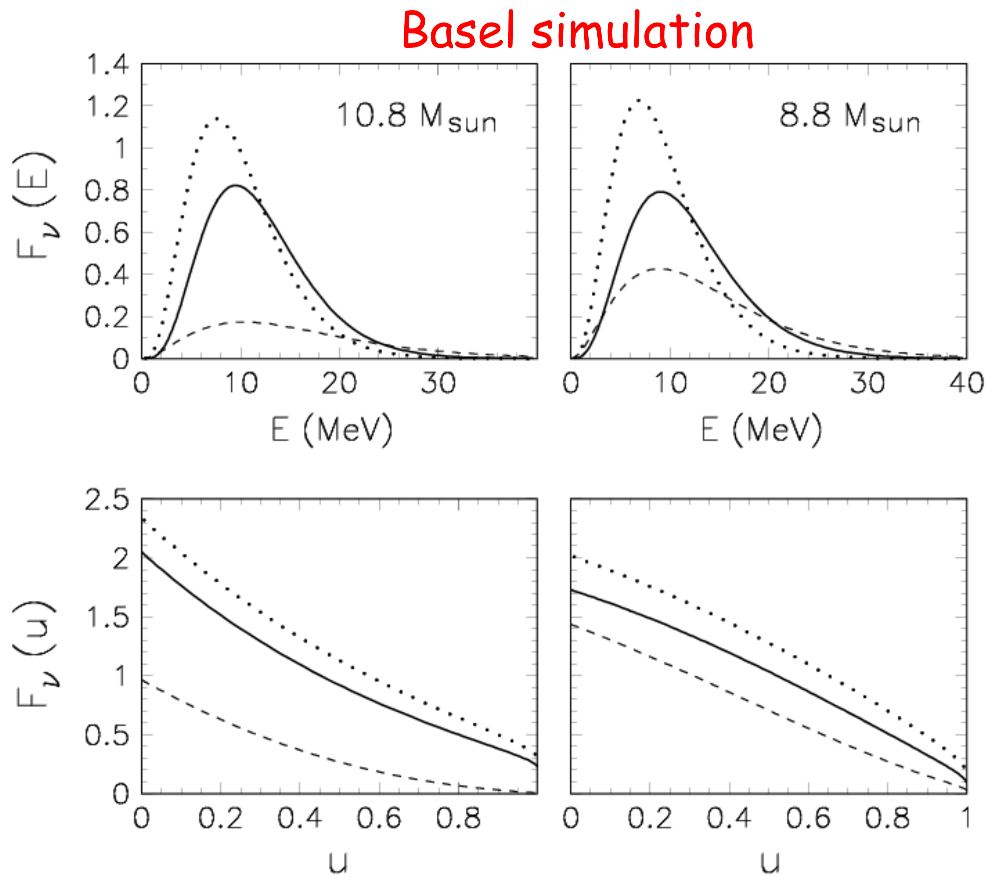
NO COLLECTIVE OSCILLATIONS DURING THE ACCRETION PHASE

Schematic single-energy (E=15 MeV) multi-angle treatment (half-isotropic emission)



REALISTIC ENERGY AND ANGULAR SPECTRA

[*Saviano, Chakraborthy, Fischer, A.M., 1203.1484*]



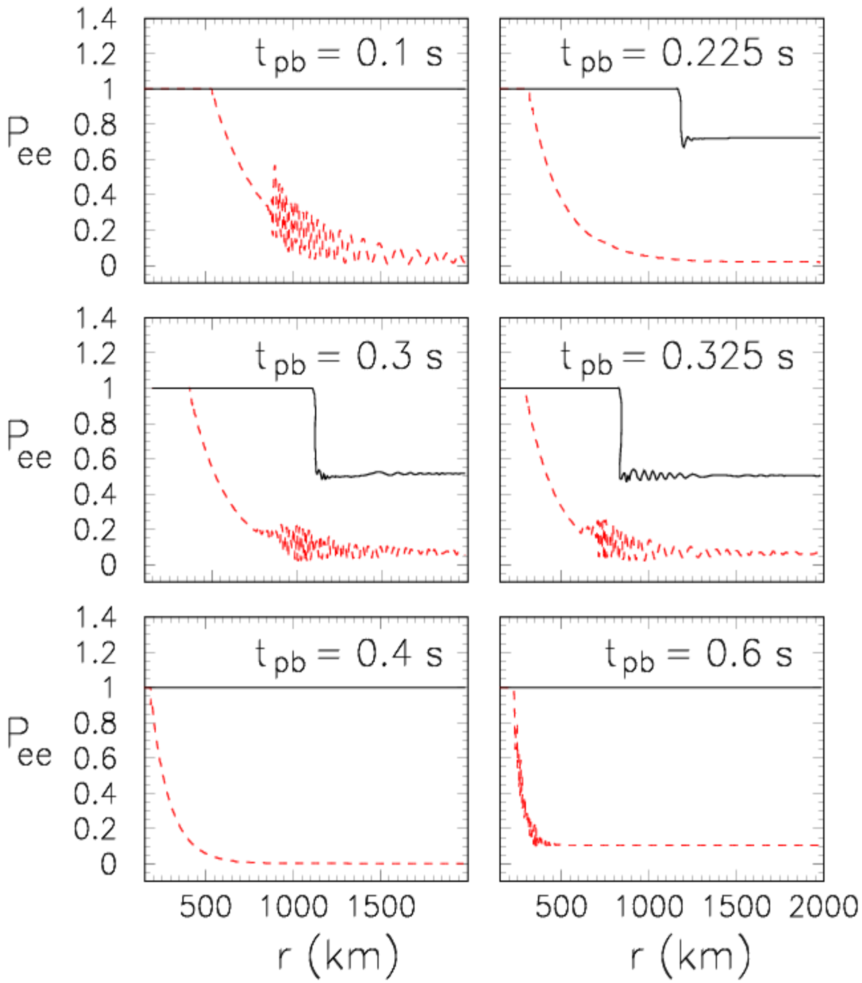
- Do realistic energy and angular distributions affect the previous results? Multi-angle & multi-energy simulations are numerically demanding.
- Stability analysis can help to overcome these difficulties

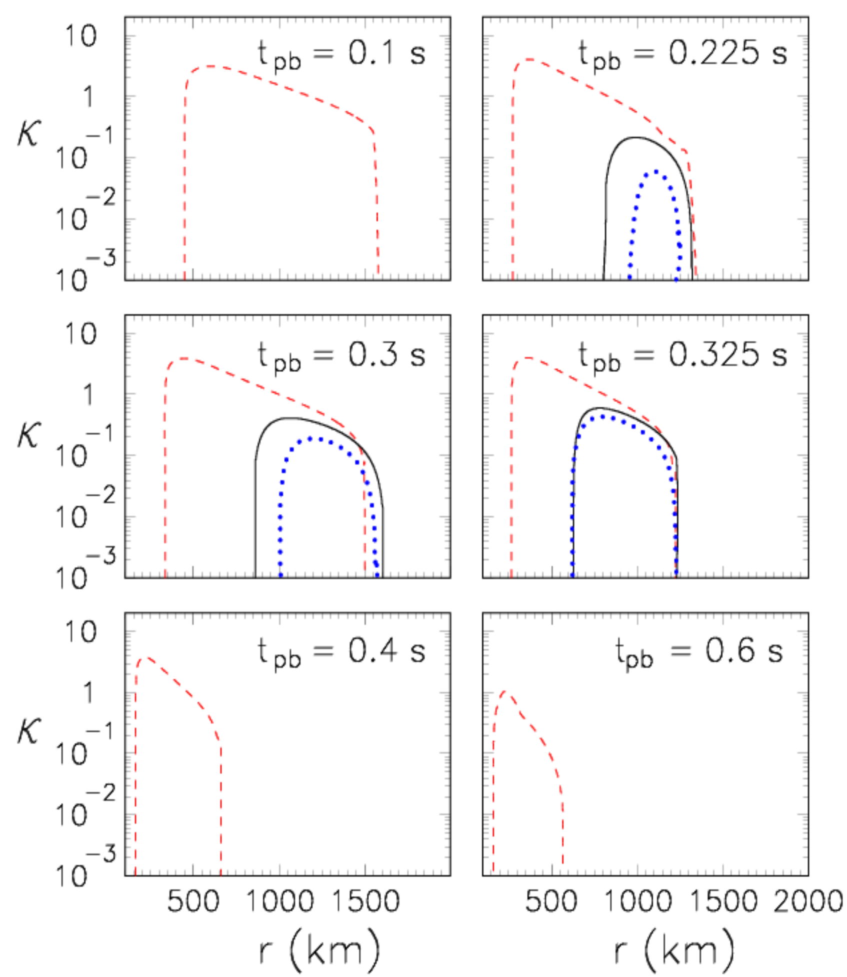
MULTI-ANGLE SIMULATIONS VS STABILITY ANALYSIS

Chakraborty et al., 1105.1130

10.8 Msun

Saviano et al., 1203.1484

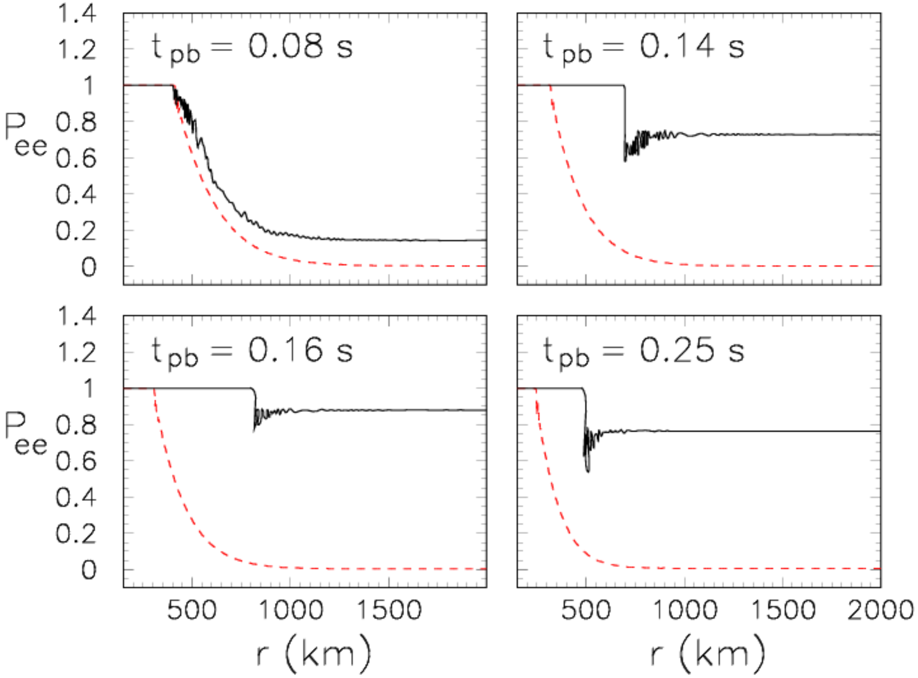


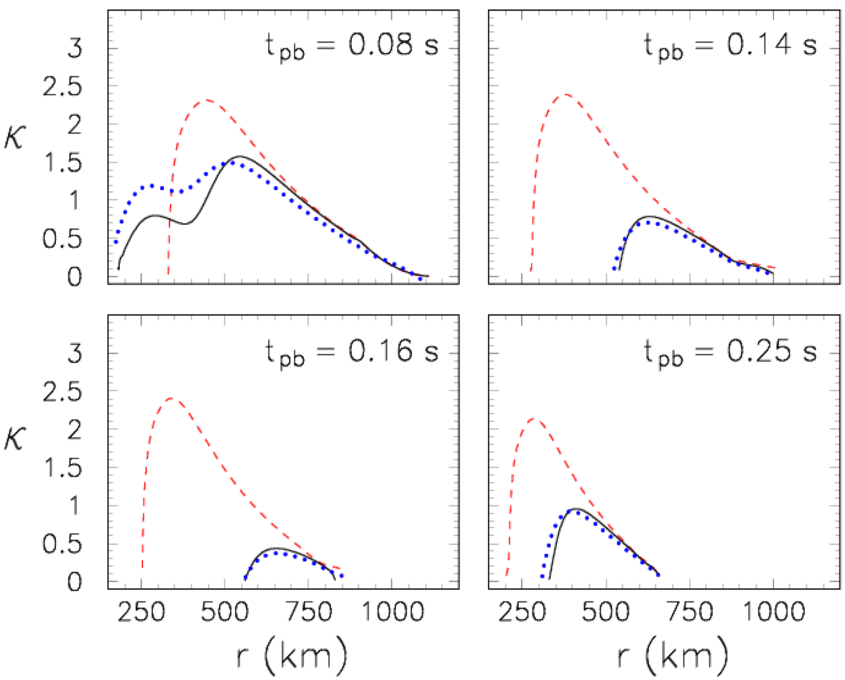


- Single energy (E=15 MeV)
- half-isotropic emission

Realistic energy spectra
- $\lambda=0$
- $\lambda\neq 0$ half-isotropic emission
- $\lambda\neq 0$ realistic angular distributions

Chakraborty et al., 1105.1130 8.8 Msun *Saviano et al., 1203.1484*





- For low-mass O-Ne-Mg SNe $n_e \sim n_\nu$ → Matter suppression is never complete.

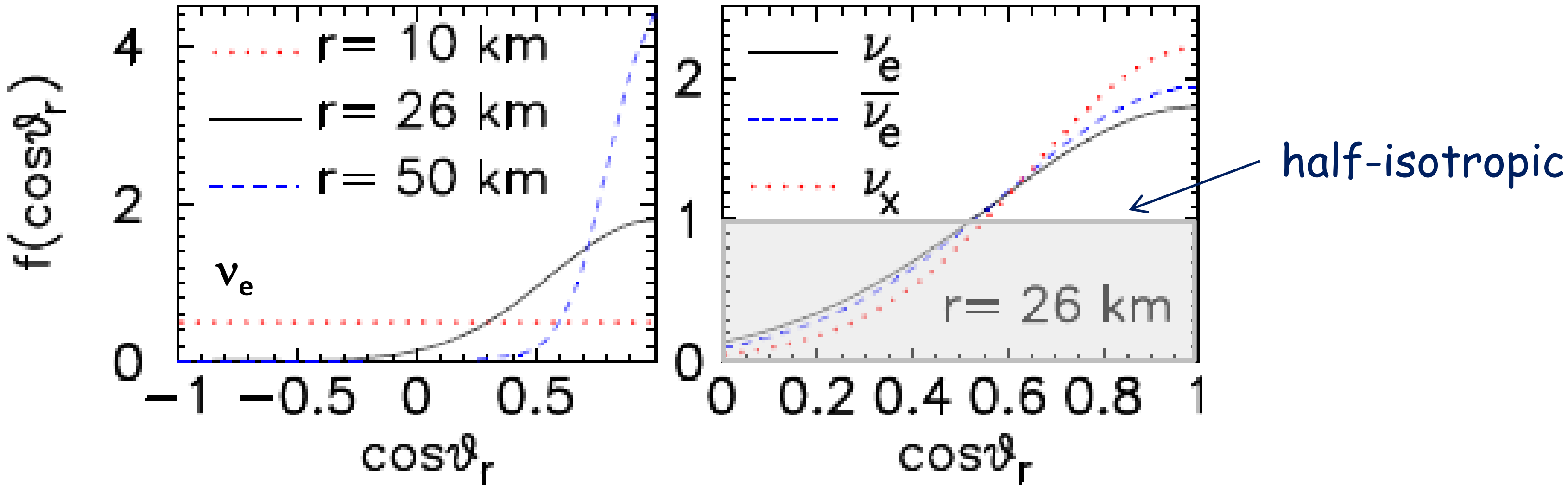
- In general, stability analysis confirms results found with multi-angle simulations

- Accurate prediction for the onset of the conversions

[Stability analysis performed with Garching SN models in *Sarikas et. al., 1109.3601; 1204.0971]*

FLAVOR-DEPENDENT NEUTRINO ANGULAR DISTRIBUTIONS

10.8 M_{sun} simulation at t = 1s (cooling) by Basel group
[*Fischer et al. arXiv: 0908.1871*]



Half-isotropic ν emission (as in bulb model) from a common neutrinosphere is not a good description. Neutrinos emerge from a thick layer.

Fix a conventional neutrinosphere where the backward flux is negligible. Neutrino angular distributions will be fwd-peaked and flavor dependent.

HOW DOES IT AFFECT THE NEUTRINO EVOLUTION?

BEYOND THE BULB MODEL

[A.M., P. Serpico, 1110.0022 (PRL), 1208.0157]

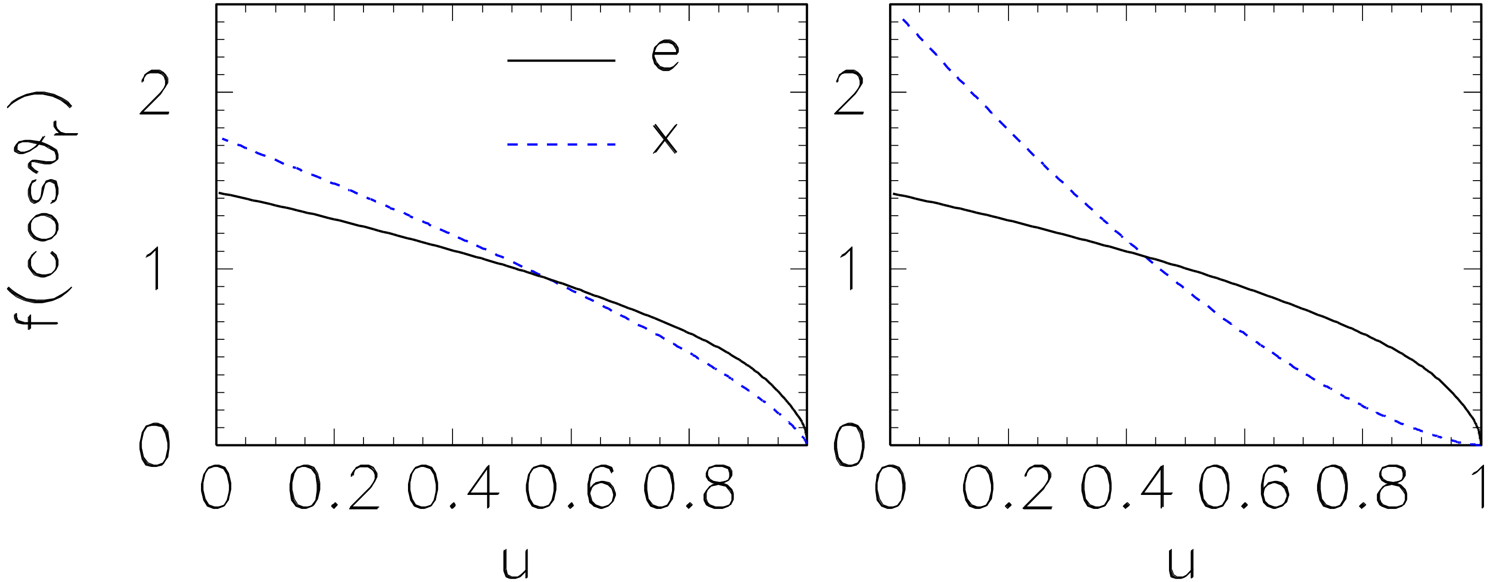
We schematically assume flavor-dependent forward-peaked distributions

$$U_{\nu_\alpha}(u) \propto (1-u)^{\beta_\alpha/2}$$

where $u = \sin^2 \theta_R$ and θ_R is the emission angle at the neutrinospehere r=R

We take $U_{\nu_e} = U_{\bar{\nu}_e}$

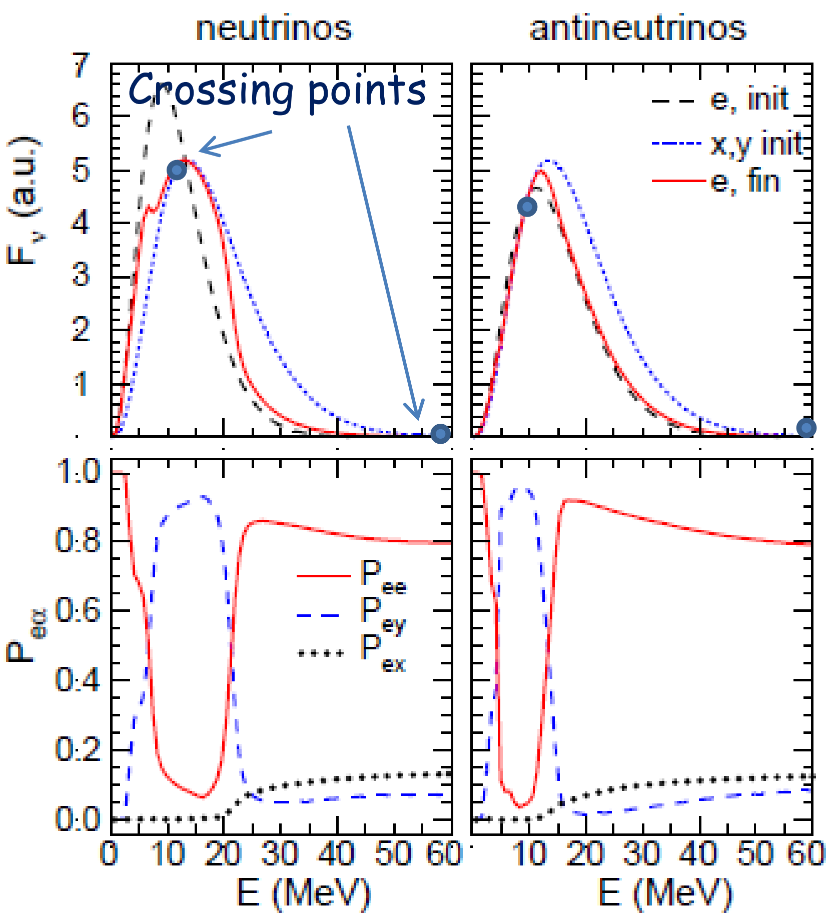
$(\beta_e = 1.0, \beta_x = 1.5)$ $(\beta_e = 1.0, \beta_x = 3.0)$



CROSSING POINTS IN THE ENERGY SPECTRA

Spectral splits triggered by instabilities around the crossing points of the original ν spectra *[Raffelt, 0810.1407, Dasgupta et al., 0904.3542]*

[A.M. & Tomas, 1012.1339]



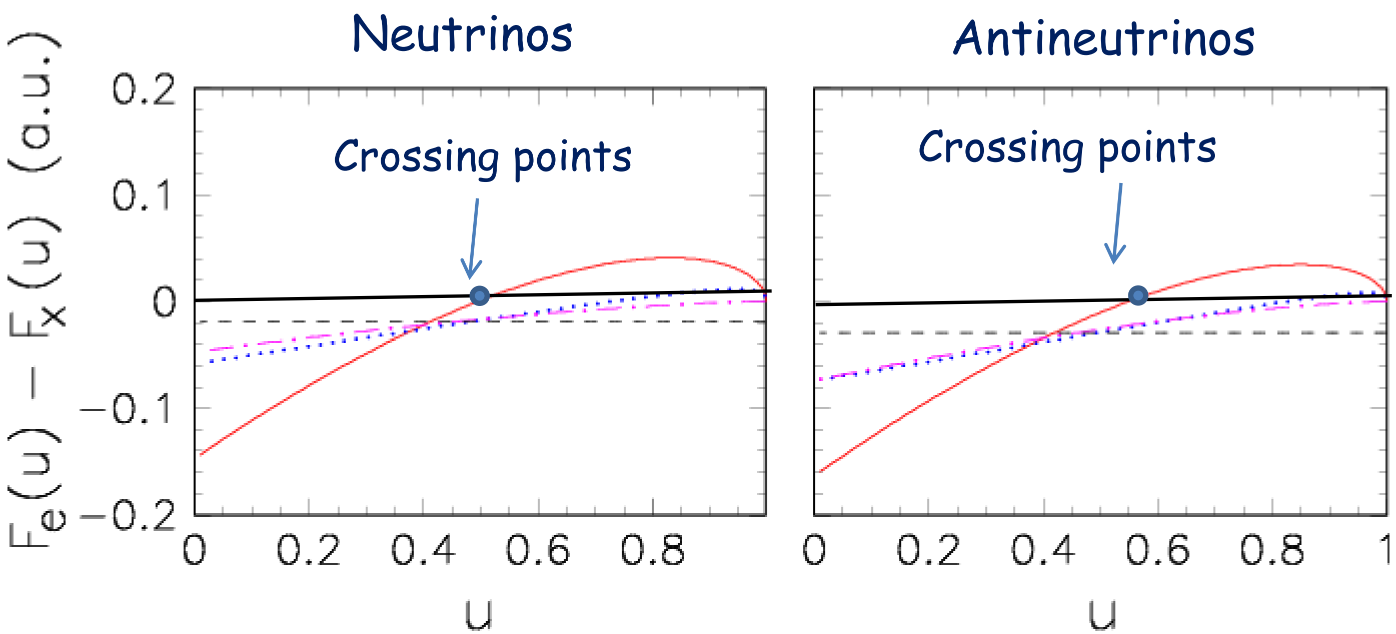
$$F_{\nu e} : F_{\bar{\nu} e} : F_{\nu x} = 0.85 : 0.75 : 1.00$$

Multiple crossing points

⇓

Multiple spectral splits

CROSSING POINTS IN THE ANGULAR SPECTRA



- - - - - $(\beta_e = 0.0, \beta_x = 0.0)$ half-isotropic

··········· $(\beta_e = 1.0, \beta_x = 1.5)$

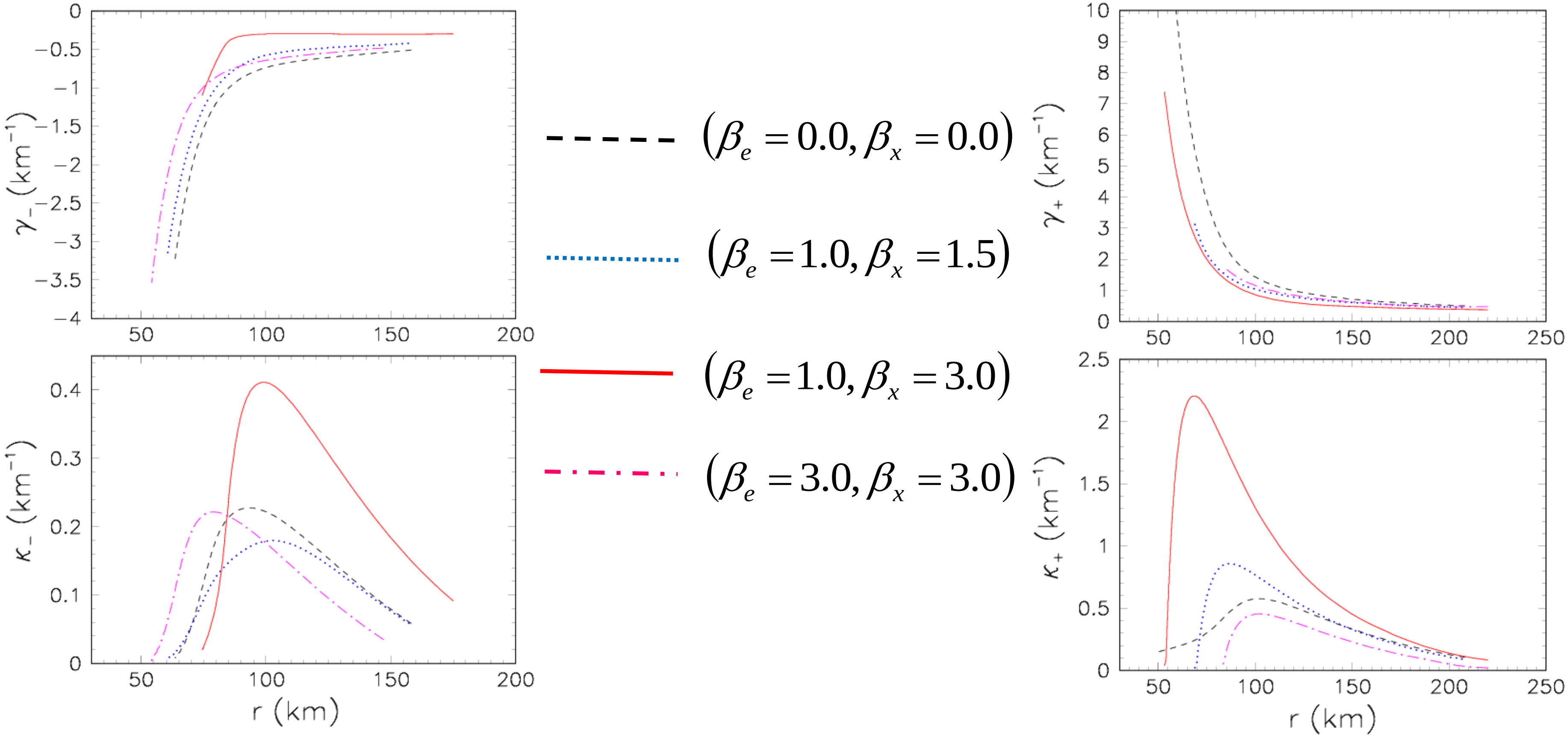
——— $(\beta_e = 1.0, \beta_x = 3.0)$

-·-·-·- $(\beta_e = 3.0, \beta_x = 3.0)$

The presence of crossing points in the energy-integrated angular spectra [F_e (u) = F_x (u)] can induce a new multi-angle instability.

FLAVOR STABILITY ANALYSIS

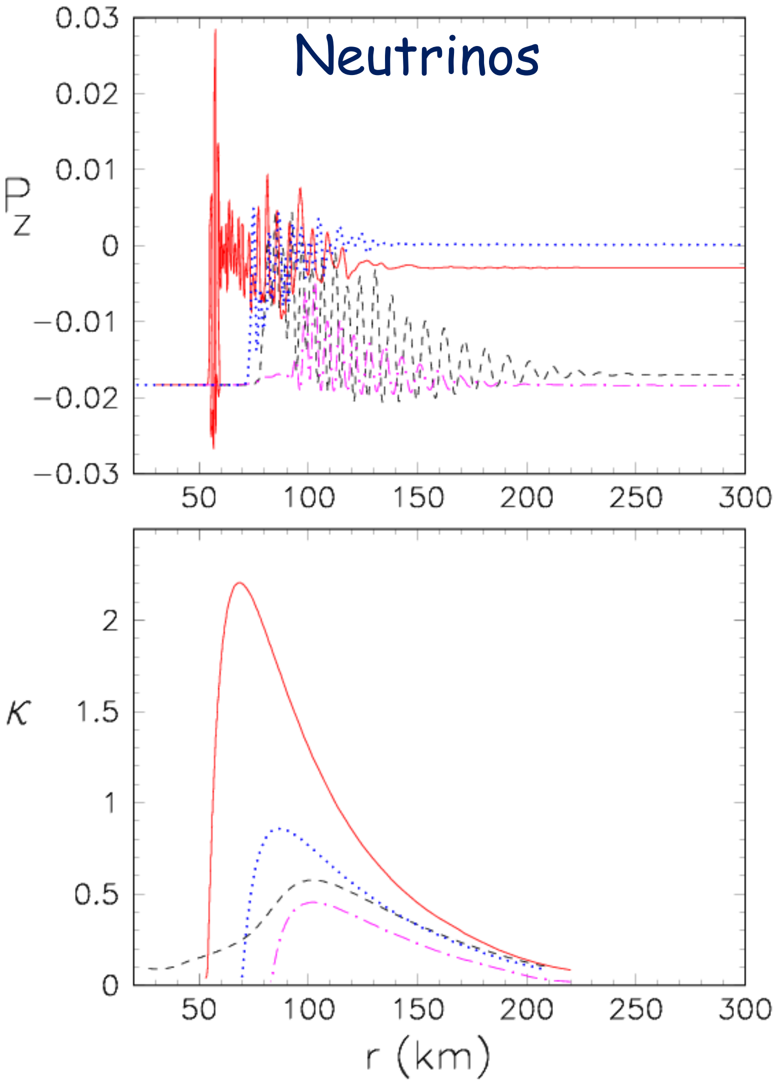
[A.M., P. Serpico, 1110.0022 (PRL), 1208.0157]



The eigenvalue equation admits a pair of solutions (γ_- , κ_-) and (γ_+ , κ_+). However, it always results $\kappa_+ > \kappa_-$.

A NEW MULTI-ANGLE INSTABILITY

[A.M., P. Serpico, 1110.0022 (PRL), 1208.0157]



Stability analysis gives the correct onset of the conversions

- - - - - $(\beta_e = 0.0, \beta_x = 0.0)$ half-isotropic

Onset at r~ 70 km

.... $(\beta_e = 1.0, \beta_x = 1.5)$

Onset at r~ 70 km as in the half-isotr. case. Enhancement of the instability. After that, strong differences. $P_z \approx 0$ ($F_e \sim F_x$) : similar oscillated spectra.

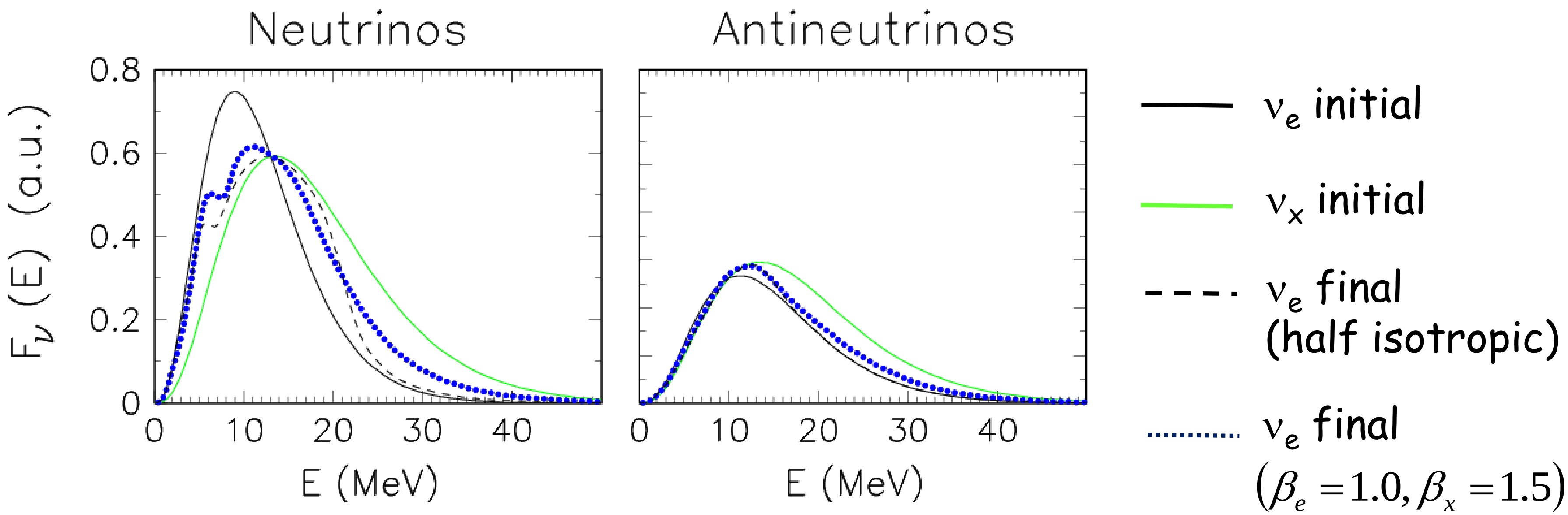
——— $(\beta_e = 1.0, \beta_x = 3.0)$

Earlier onset at r~ 55 km. Strong enhancement of the instability. Final $F_e \sim F_x$

-·-·-· $(\beta_e = 3.0, \beta_x = 3.0)$

Flavor-blind distributions. Suppression of the instability. Only minor deviations wrt to the half-isotropic case.

MULTI-ANGLE EFFECTS IN THE COOLING PHASE



- Spectral splits found in the half-isotropic case are smeared-out and swaps are not complete.
- Necessary deviations from flavor-blind distributions to trigger large effects
- No major effects for the accretion spectra (no crossing point in the angular spectra)

CONCLUSIONS

Multi-angle effects in self-induced SN ν flavor conversions can exhibit a rich phenomenology, with a strong dependence on ν energy and angular spectra which significantly vary during the ν emission.

- Brute force numerical simulations of the multi-angle flavor evolution are time consuming.
- Linearized stability analysis of EOMs can simplify this problem determining under which condition the ν system is stable or unstable.
- Two applications: matter-induced suppression of self-induced conversions (accretion phase), multi-angle instability associated with flavor-dependent angular distributions (cooling phase).
- Once unstable cases are identified, large scale numerical simulations are mandatory to determine the final SN ν spectra
- Suprises can still emerge, e.g. would the removal of the azimuthal symmetry trigger a new instability?

LOT OF NUMERICAL AND ANALYTICAL WORK STILL NECESSARY!