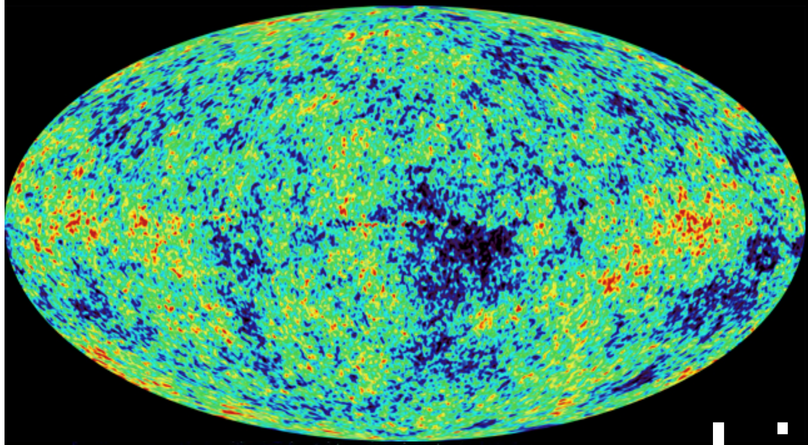
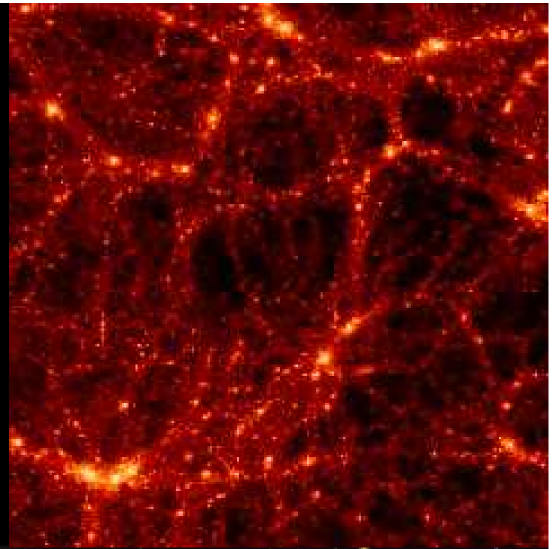
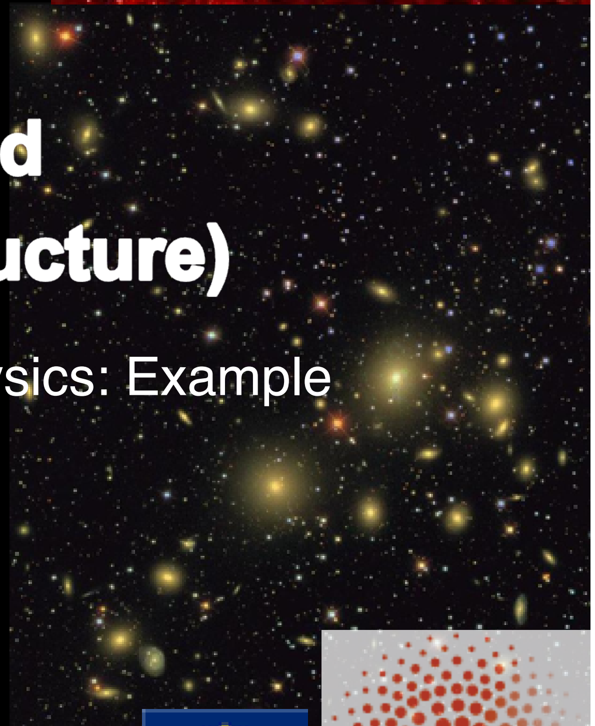




Licia Verde

Neutrino properties and cosmology (large-scale structure)

Connecting Cosmology to Fundamental Physics: Example



http://icc.ub.edu/~liciaverde

Overview

Recent work on neutrino properties from cosmology

Neutrino masses

Number of families

Mass hierarchy

MODEL dependence…

New developments: data

CMB damping tail (ACT, SPT)

Sloan Digital Sky Survey BOSS, WIGGLEZ

Baryon Acoustic Oscillations & clustering

Direct measurement of expansion history

NEWS: FUTURE DATA: Euclid, recycled spy satellite, WFIRST

New developments: theory

Better modeling of non-linearities via N-body simulations (and perturbation theory)

Neutrino mass: Physical effects

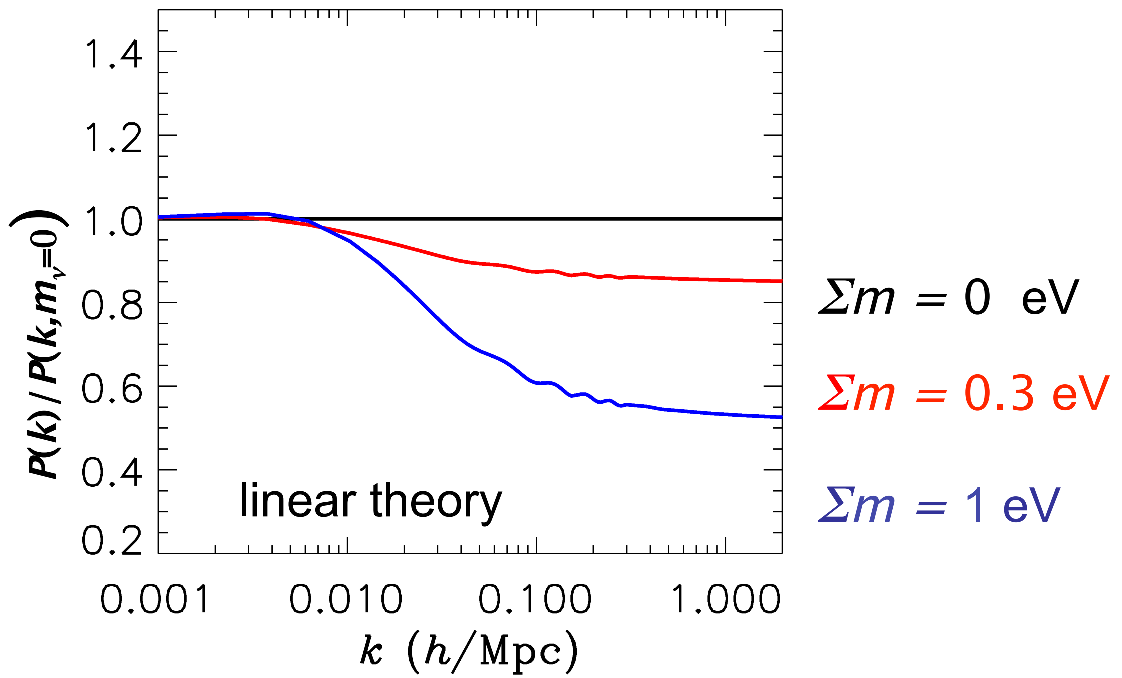
Total mass >~1 eV become non relativistic before recombination CMB

Total mass <~1 eV become non relativistic after recombination:
alters matter-radn equality but effect can be "cancelled"
by other parameters

Degeneracy

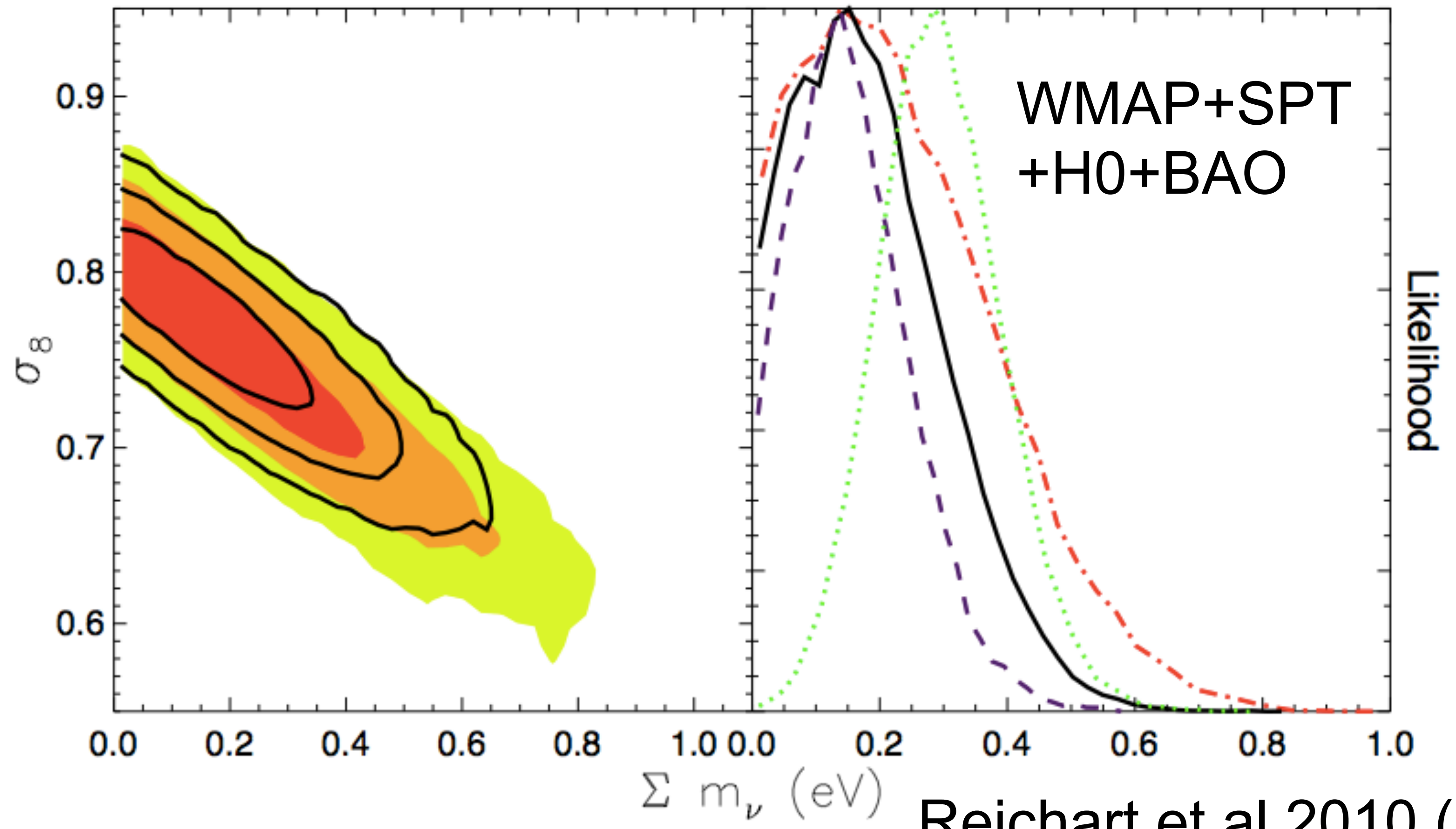
After recombination

FINITE NEUTRINO MASSES SUPPRESS THE MATTER POWER SPECTRUM ON SCALES SMALLER THAN THE FREE-STREAMING LENGTH



WMAP: $\Sigma < 1.3$ eV (95% CL)

Adding measurements of the CMB damping tail to the primary signal (WMAP) does not help much (look into secondary effects)



Reichart et al 2010 (SPT)

Must add low redshift Universe information (s8, clustering etc.)

Robust neutrino constraints on the total mass

Reid, et al 2010, WMAP+H0+MaxBCG, SDSS DR7 P(k)

$\sigma_8(\Omega_m/0.25)^{0.41} = 0.832 \pm 0.033.$

	Bayesian	upper 95% C.L. bound on $\sum m_\nu$			
model	base dataset	–	+maxBCG	$+H_0$	+maxBCG+H_0
ΛCDM	WMAP5	1.3	1.1	0.59	0.40
ΛCDM	WMAP5+BAO+SN	0.67	0.35	0.59	0.31
ΛCDM $+\alpha$	WMAP5	1.34	1.25	0.54	0.39
ΛCDM $+r$	WMAP5	1.36	1.18	0.83	0.40
wCDM	WMAP5+BAO+SN	0.80	0.52	0.72	0.47

Also: Thomas et al. (2010), Gonzalez-Garcia et al (2010), Giusarma et al (2011), Riemer-Sørensen (2012, wigglez++) etc.

dePutter et al 2012 (SDSS DR9 BOSS)

95% CL $\sum m_\nu$[eV]	prior only	prior+CMASS,$\ell_{\max} = 150$	prior+CMASS,$\ell_{\max} = 200$
WMAP7 prior	1.1	0.74 (0.92)	0.56 (0.90)
WMAP7 + HST prior	0.44	0.31 (0.40)	0.26 (0.36)

TABLE 1

THE 95% CONFIDENCE LEVEL UPPER LIMITS ON THE SUM OF THE NEUTRINO MASSES Σm_ν. THE TOP ROW INVESTIGATES THE EFFECT OF ADDING THE CMASS GALAXY POWER SPECTRA TO A WMAP PRIOR WHILE THE BOTTOM ROW USES WMAP AND THE H_0 CONSTRAINT FROM HST AS A PRIOR. IN PARENTHESES WE SHOW RESULTS FOR THE MORE CONSERVATIVE MODEL MARGINALIZING OVER THE SHOT NOISE-LIKE PARAMETERS a_i.

Robust neutrino constraints… on the total mass

Reid, et al 2010, WMAP+H0+MaxBCG, SDSS DR7 P(k)

$\sigma_8(\Omega_m/0.25)^{0.41} = 0.832 \pm 0.033.$

Bayesian		upper 95% C.L. bound on $\sum m_\nu$			
model	base dataset	–	+maxBCG	$+H_0$	+maxBCG+H_0
ΛCDM	WMAP5	1.3	1.1	0.59	0.4[illegible]
ΛCDM	WMAP5+BAO+SN	0.67	0.35	0.59	[illegible]
ΛCDM +α	WMAP5	1.34	1.25	[illegible]	[illegible]
ΛCDM +r	WMAP5	1.36	1.1[illegible]	[illegible]	[illegible].40
wCDM	WMAP5+BAO+SN	0.80	[illegible]	[illegible]	0.47

Σ<0.3 (95%CL) in a minimal LCDM scenario

Also: Thomas et al. (2010), C[illegible] et al (2010), Giusarma et al (2011)
Riemer-Sørensen (20[illegible] etc.

dePutter [illegible] (SDSS DR9 BOSS)

[illegible]% CL $\sum m_\nu$[eV]	prior only	prior+CMASS,$\ell_{\max} = 150$	prior+CMASS,$\ell_{\max} = 200$
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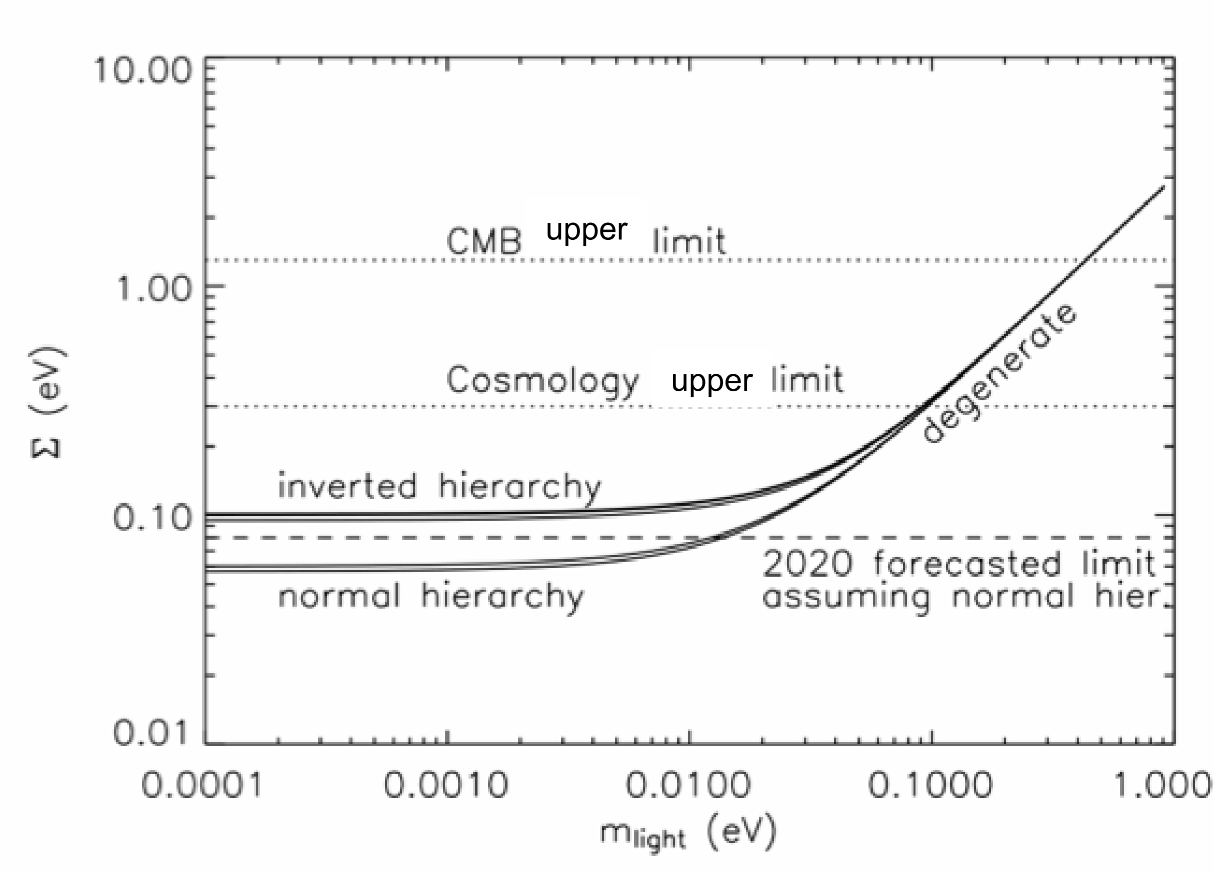
Outlook towards the future

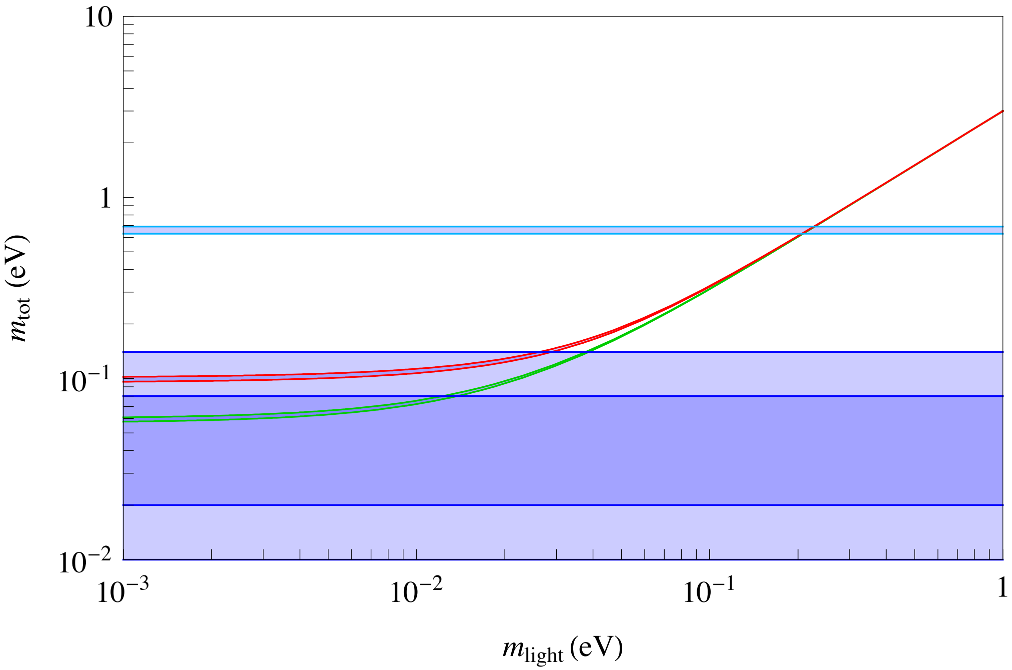


The future is bright: forecasts

Detailed errors depend on what assumptions about underlying cosmology one is willing to make

Example:

LSST (lensing)
+Planck primary



Look how errors depend on fiducial model

More forecasts: space-based

Carbone, LV et al (2012)

Two surveys strategies

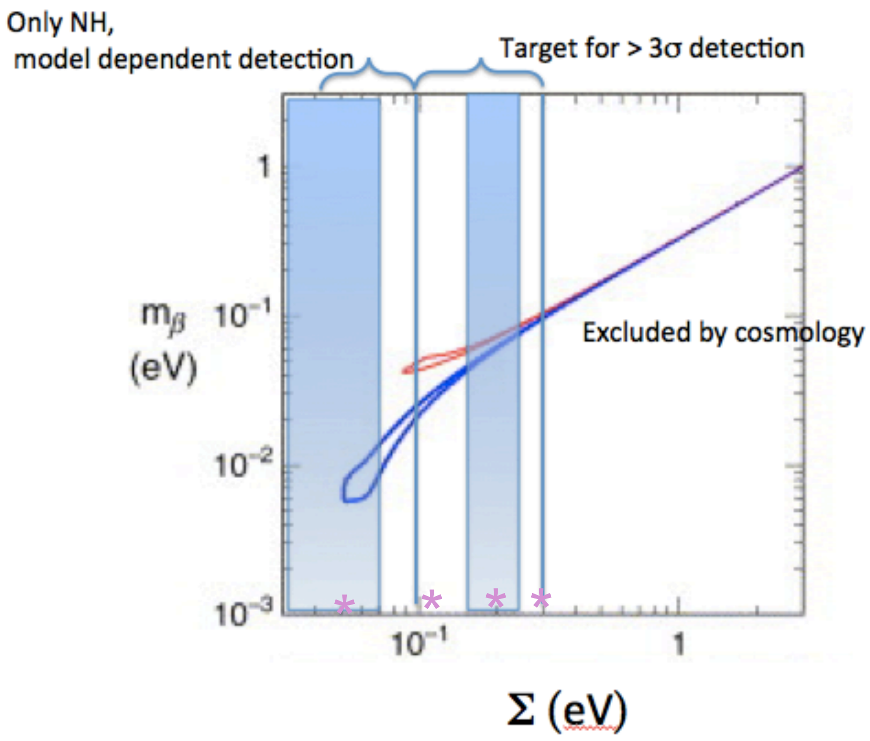
A EUCLID like survey (slitless spectroscopy of $H\alpha$ emission lines)

H-band magnitude limited survey , multislit spectroscopy (WFIRST)

Include redshift space distortions,
signal from growth,
different baseline models, etc.

Several fiducial models *

Study parameters degeneracies and
Survey set up and performance



Errors depend on fiducial model

More forecasts: space-based

Carbone, LV et al (2012)

Two surveys strategies

A EUCLID like survey (slitless spectroscopy of Hα emission lines)

H-band magnitude limited survey , multislit spectroscopy (WFIRST)

Table 5: $\sigma(M_\nu)$ marginalised errors from LSS+CMB

fiducial→	M_ν=0.3 eV[a]	M_ν=0.2 eV[a]	M_ν=0.125 eV[b]	M_ν=0.125 eV[c]	M_ν=0.05 eV[b]
General cosmology					
slitless+BOSS+Planck	0.035	0.043	0.031	0.044	0.053
multi-slit+Planck	0.030	0.038	0.027	0.039	0.046
ΛCDM cosmology					
slitless+BOSS+Planck	0.017	0.019	0.017	0.021	0.021
multi-slit+Planck	0.015	0.016	0.014	0.018	0.018

[a]for degenerate spectrum: $m_1 \approx m_2 \approx m_3$; [b]for normal hierarchy: $m_3 \neq 0$, $m_1 \approx m_2 \approx 0$
[c]for inverted hierarchy: $m_1 \approx m_2$, $m_3 \approx 0$;

Beware of systematics!!!!!

It would be of great value to have an internal consistency check
(more later)

What about non-linearities?

Approaches: Analytic i.e. Perturbation theory e.g., Saito et al.

N-body Simulations Bandbyge, Hannestad et al.
Viel, Springel et al.

Intermediate: Agarval &Feldman

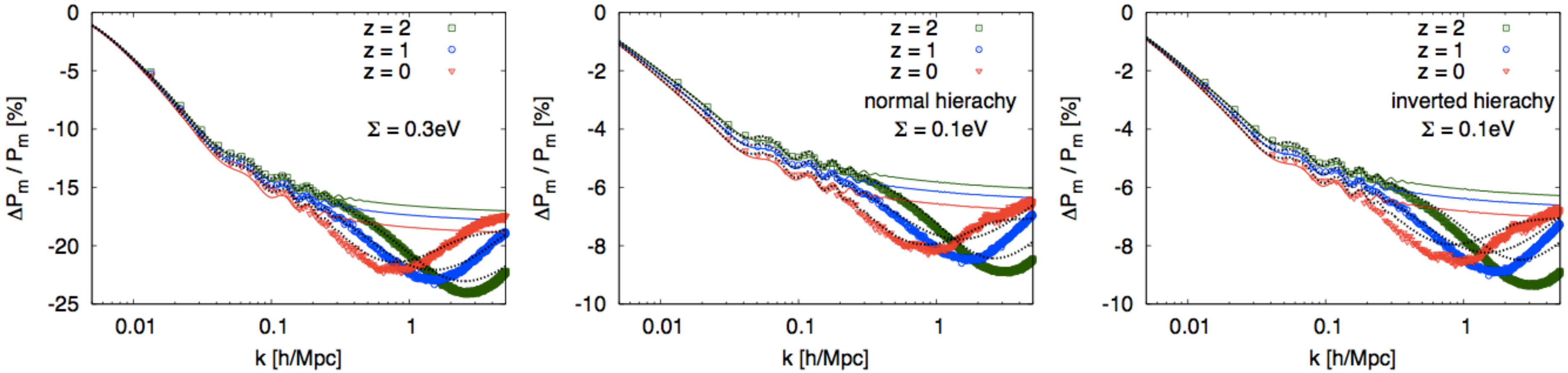
Options:

Simulate just neutrino masses

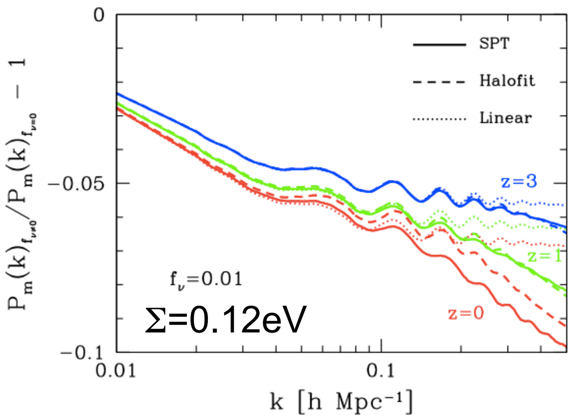
- Use particles
- Use grids
- Use hybrid

Simulate also hierarchy

Effect of Σ (total mass)



Wagner Verde Jimenez 2012



Saito et al 09

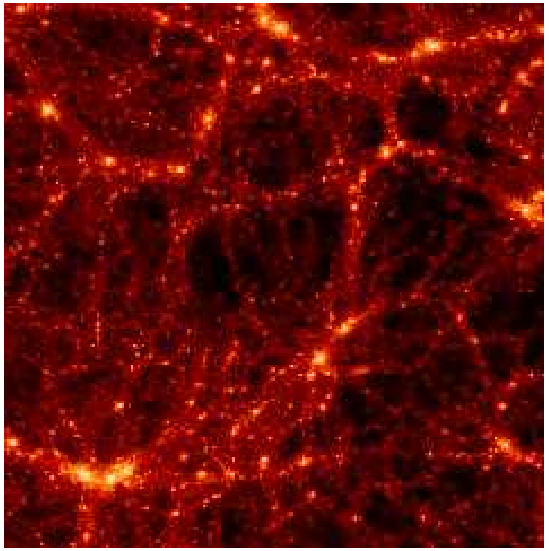
This is for MATTER in real space

Enhancement of the effect on interesting scales!

What about real world effects?

- Baryonic physics (lensing and galaxy surveys)
- Bias (galaxy surveys)
- redshift space (galaxy surveys)

Recent work (Audren et al arXiv:1210.2194) indicate that the last two might be limiting effect to use the non-linear information



$N_{eff:}$ number of effective species

$$H^2(t) \simeq \frac{8\pi G}{3}(\rho_\gamma + \rho_\nu) \qquad \rho_\nu \propto T^4 N_{\rm eff}$$

Standard: N_{eff}=3.045

Any thermal background of light particles, anything affecting expansion rate

Look at BBN Neff around 3 to 4

Systematics!

Nollett, Holder 2011: Yp difficult, better use CMB ($\Omega_b h^2$)+D/H

Pettini, Cooke 2012 $\mathrm{N}_\nu = 3.0 \pm 0.5$

N_{ν}: number of effective species

$$H^2(t) \simeq \frac{8\pi G}{3}(\rho_\gamma + \rho_\nu) \qquad \rho_\nu \propto T^4 N_{\rm eff}$$

Standard: N_{eff}=3.045

Any thermal background of light particles, anything affecting expansion rate

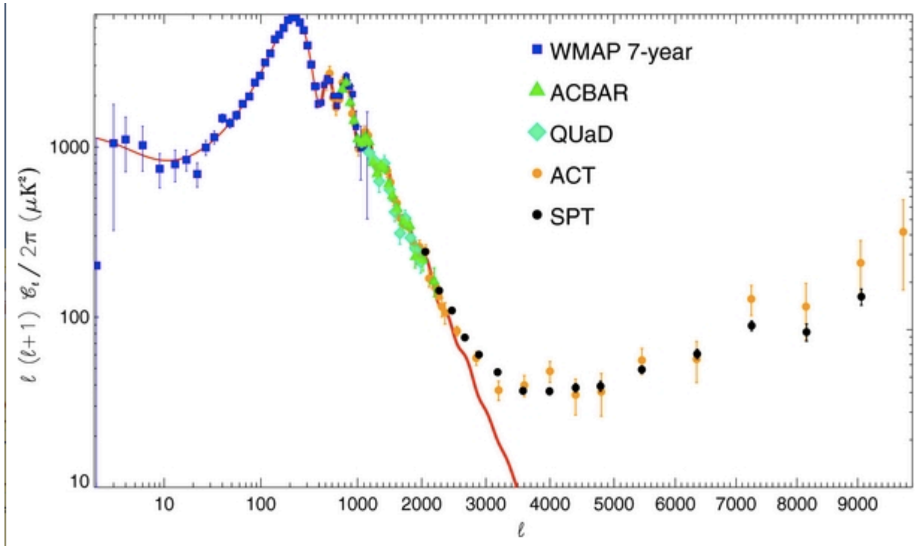
Look at BBN

Neff= 3 to 4

Systematics!

Look at CMB:
effects matter-radn equality
and so sound horizon at decoupling
-> degeneracy with ω_m and H

Anisotropic stress,
z_{eq} on diffusion damping



Literature review

Cosmological analyses consistently give best fit values >3.04.
“dark radiation”
But analyses are NOT independent
(WMAP is always in common, H0 many times in common)

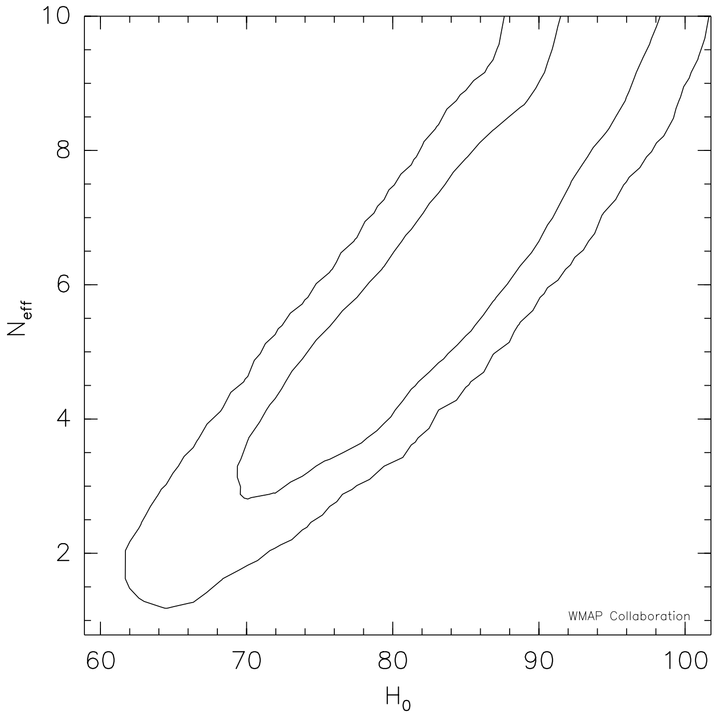
It’s barely 2 sigmas (except for one data set: ACT)

Also,
beware of degeneracies

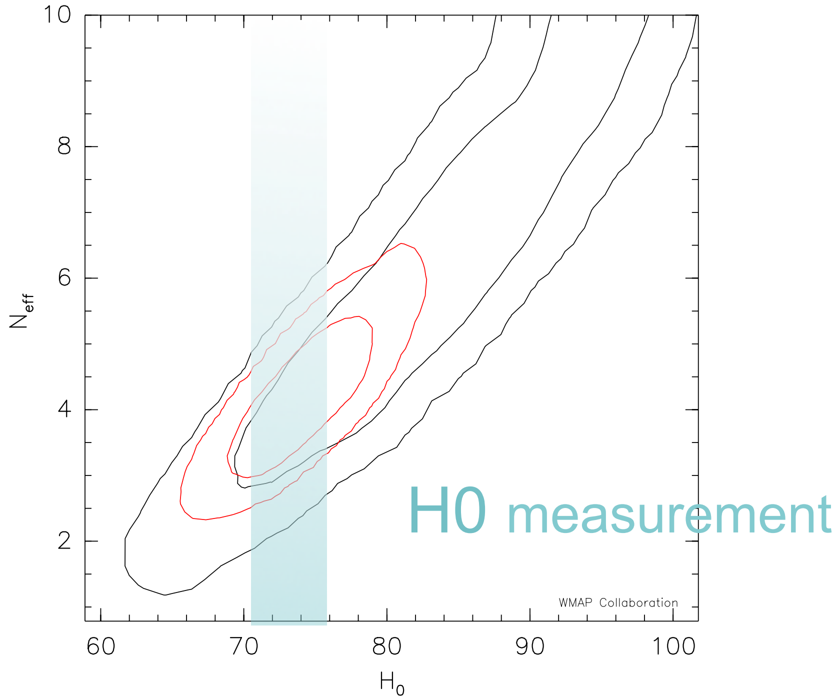
Tab 3 white paper
1204.5379

Model	Data	N_{eff}	Ref.	
N_{eff}	W-5+BAO+SN+H_0	$4.13^{+0.87(+1.76)}_{-0.85(-1.63)}$	[346]	Reid et al ‘10
	W-5+LRG+H_0	$4.16^{+0.76(+1.60)}_{-0.77(-1.43)}$	[346]	
	W-5+CMB+BAO+XLF+f_{gas}+H_0	$3.4^{+0.6}_{-0.5}$	[349]	Mantz et al 10
	W-5+LRG+maxBCG+H_0	$3.77^{+0.67(+1.37)}_{-0.67(-1.24)}$	[346]	Reid et al ‘10
	W-7+BAO+H_0	$4.34^{+0.86}_{-0.88}$	[338]	Komatsu et al 11
	W-7+LRG+H_0	$4.25^{+0.76}_{-0.80}$	[338]	(WMAP7)
	W-7+ACT	5.3 ± 1.3	[343]	Dunkley et al 10
	W-7+ACT+BAO+H_0	4.56 ± 0.75	[343]	(ACT)
	W-7+SPT	3.85 ± 0.62	[344]	Keisler et al 11
	W-7+SPT+BAO+H_0	3.85 ± 0.42	[344]	(SPT)
	W-7+ACT+SPT+LRG+H_0	$4.08^{(+0.71)}_{(-0.68)}$	[350]	Archidiacono et al 20
	W-7+ACT+SPT+BAO+H_0	3.89 ± 0.41	[351]	
$N_{eff}+f_\nu$	W-7+CMB+BAO+H_0	$4.47^{(+1.82)}_{(-1.74)}$	[352]	Hamann et al 2010
	W-7+CMB+LRG+H_0	$4.87^{(+1.86)}_{(-1.75)}$	[352]	
$N_{eff}+\Omega_k$	W-7+BAO+H_0	4.61 ± 0.96	[351]	Smith et al 2011
	W-7+ACT+SPT+BAO+H_0	4.03 ± 0.45	[352]	Hamann et al 2010
$N_{eff}+\Omega_k+f_\nu$	W-7+ACT+SPT+BAO+H_0	4.00 ± 0.43	[351]	Smith et al 2011
$N_{eff}+f_\nu+w$	W-7+CMB+BAO+H_0	$3.68^{(+1.90)}_{(-1.84)}$	[352]	Hamann et al 201
	W-7+CMB+LRG+H_0	$4.87^{(+2.02)}_{(-2.02)}$	[352]	
$N_{eff}+\Omega_k+f_\nu+w$	W-7+CMB+BAO+SN+H_0	$4.2^{+1.10(+2.00)}_{-0.61(-1.14)}$	[353]	Gonzalez-Garcia
	W-7+CMB+LRG+SN+H_0	$4.3^{+1.40(+2.30)}_{-0.54(-1.09)}$	[353]	et al. 2010

What may be going on?



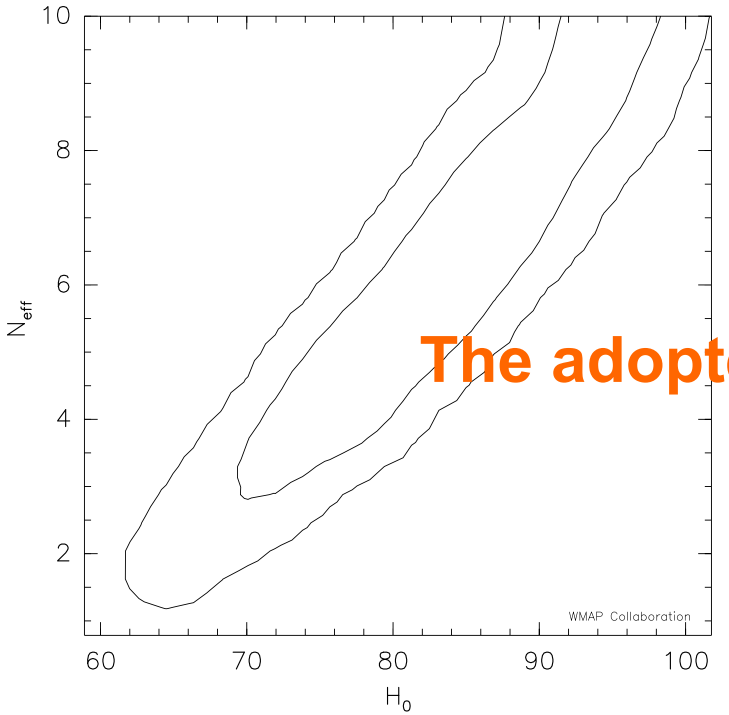
WMAP only



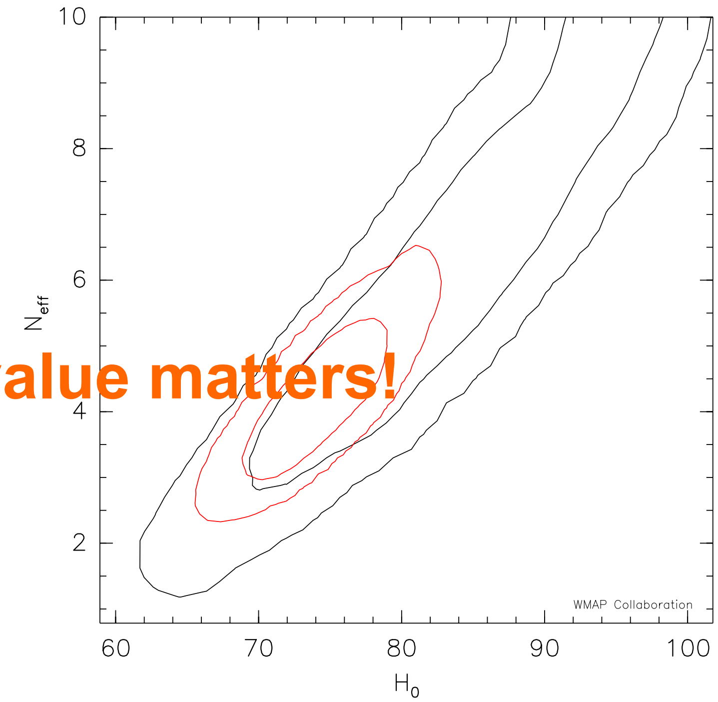
WMAP+H0+BAO

Straight from the on-line LAMBDA cosmological parameters plotter

What may be going on?



WMAP only



WMAP+H0+BAO

The adopted H0 value matters!

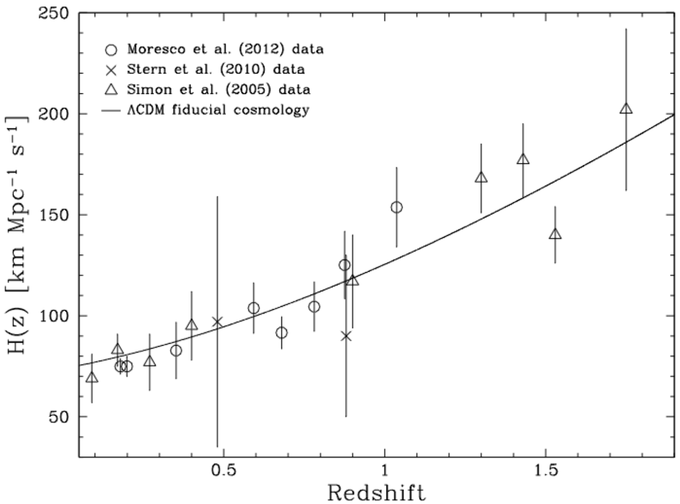
Straight from the on-line LAMBDA cosmological parameters plotter

It is worth recalling that

- For a single experiment the reported error are statistical + systematic. Hopefully systematic<statistical errors.
- When combining experiments all errors are treated as statistical! Eventually (when adding many datasets) systematic ≥ statistical! Shifts + too small error-bars

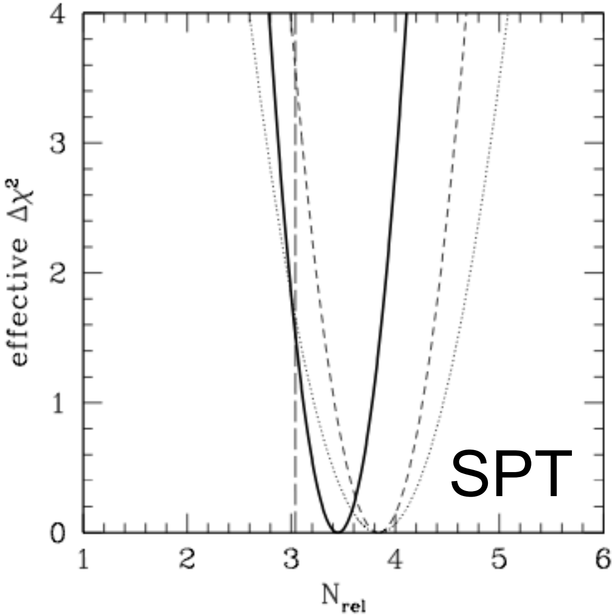
Range	Population in range	Expected frequency outside range	Approx. frequency for daily event
$\mu \pm 1\sigma$	0.682 689 492 137 086	1 in 3	Twice a week
$\mu \pm 1.5\sigma$	0.866 385 597 462 284	1 in 7	Weekly
$\mu \pm 2\sigma$	0.954 499 736 103 642	1 in 22	Every three weeks
$\mu \pm 2.5\sigma$	0.987 580 669 348 448	1 in 81	Quarterly
$\mu \pm 3\sigma$	0.997 300 203 936 740	1 in 370	Yearly
$\mu \pm 3.5\sigma$	0.999 534 741 841 929	1 in 2149	Every six years
$\mu \pm 4\sigma$	0.999 936 657 516 334	1 in 15,787	Every 43 years (twice in a lifetime)

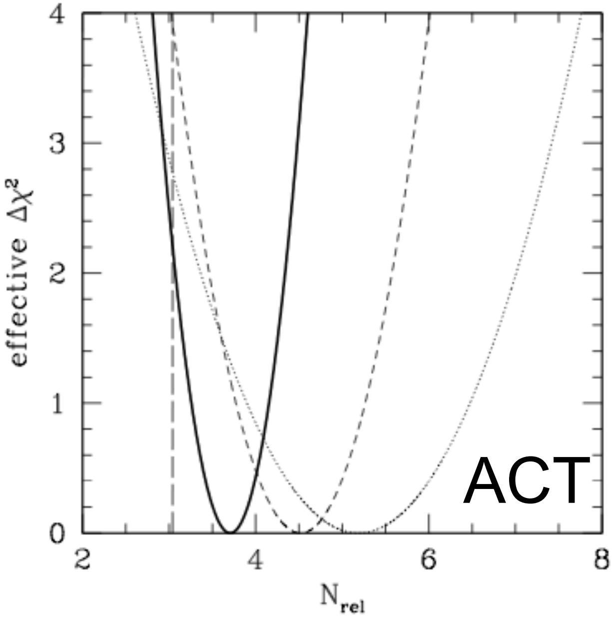
H(z) estimates



$$N_\nu = 3.59 \pm 0.48(\pm 0.94) \quad \mathrm{WMAP + ACT + H(z)}$$
$$N_\nu = 3.37 \pm 0.34(\pm 0.67) \quad \mathrm{WMAP + SPT + H(z)}$$
$$N_\nu = 3.38 \pm 0.50(\pm 1) \quad \mathrm{WMAP + H(z)}$$

$$N_\nu < 4 \text{ at } 95\%(74\%) \text{ C.L.}$$

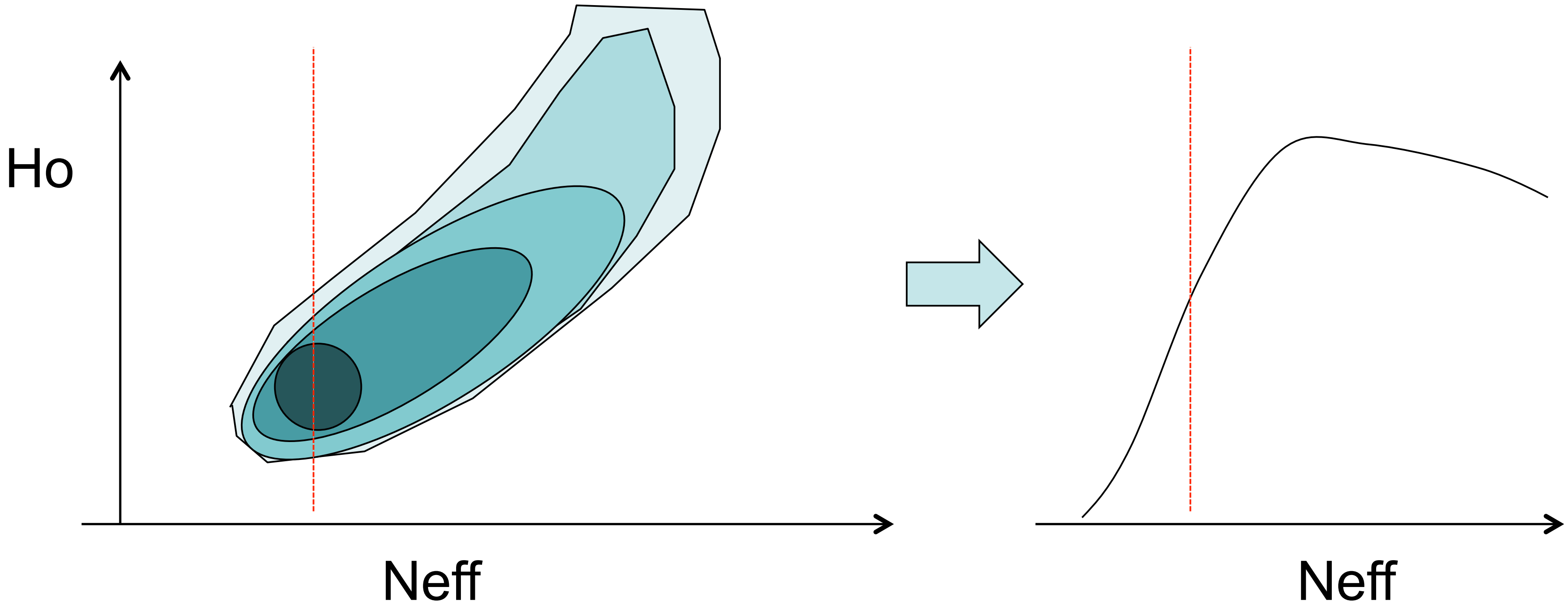




Moresco et al. 2012

Effect of priors?

"This is not interpreted as a statistically significant departure from the concordance value; the best-fit $\chi 2$ is only 1.3 less than for Neff=3.04." (from the ACT paper Dunkley et al '10)



A “frequentist”approach

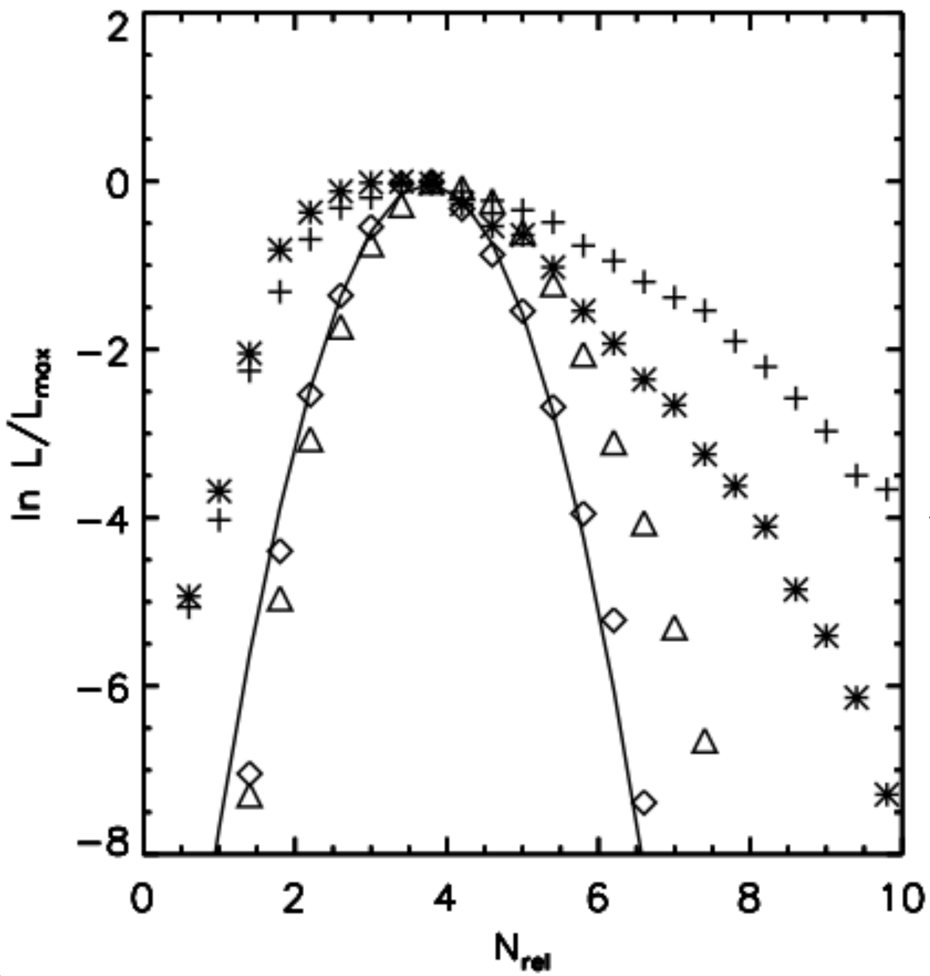
Profile likelihood ratio

Say you have n uninteresting parameters and one, θ that you are interested in e.g. $m\nu$ or Neff. For each value of θ find the maximum likelihood L_m regardless of the values assumed by the other parameters (max of the conditional likelihood). Then consider L_m/L_{max} as a function of θ.

Pros: there’s no prior in here

Cons: interpretation of confidence levels is more complex.

How does it look like



WMAP5+SDSS DR8

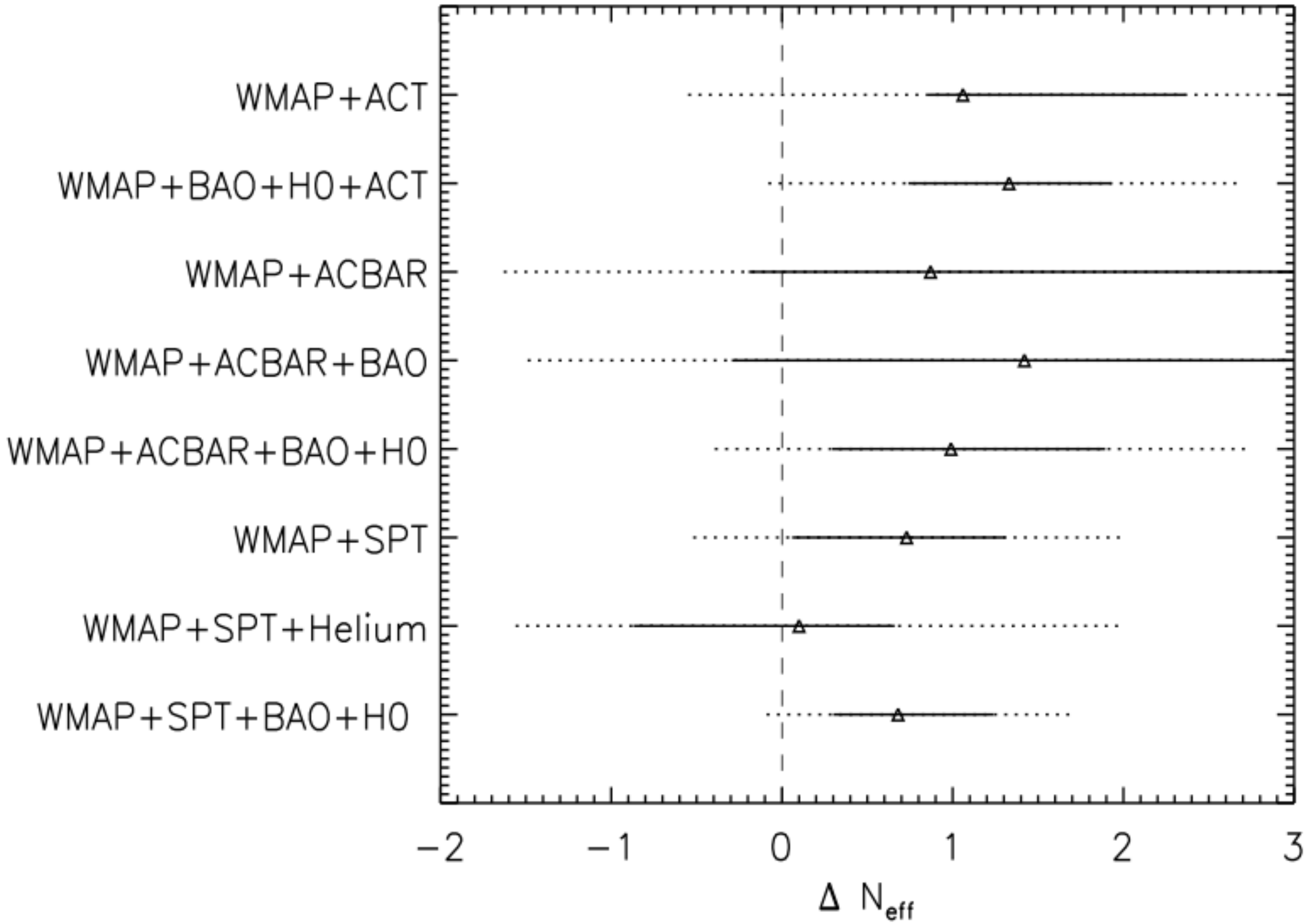
WMAP5+SDSS DR8
+maxBCG

WMAP5+SDSS DR8
+H0

WMAP5+SDSS DR8
+Hz

Reid et al 2010

Look at the likelihood



Gonzalez-Morales et al. 2011(12)?

Bayesian Evidence

$$E = \int \mathcal{L}(\theta)\Pr(\theta)d\theta \,.$$

it does not focus on the best-fitting parameters of the model, but rather asks "of all the parameter values you thought were viable before the data came along, how well on average did they fit the data?"

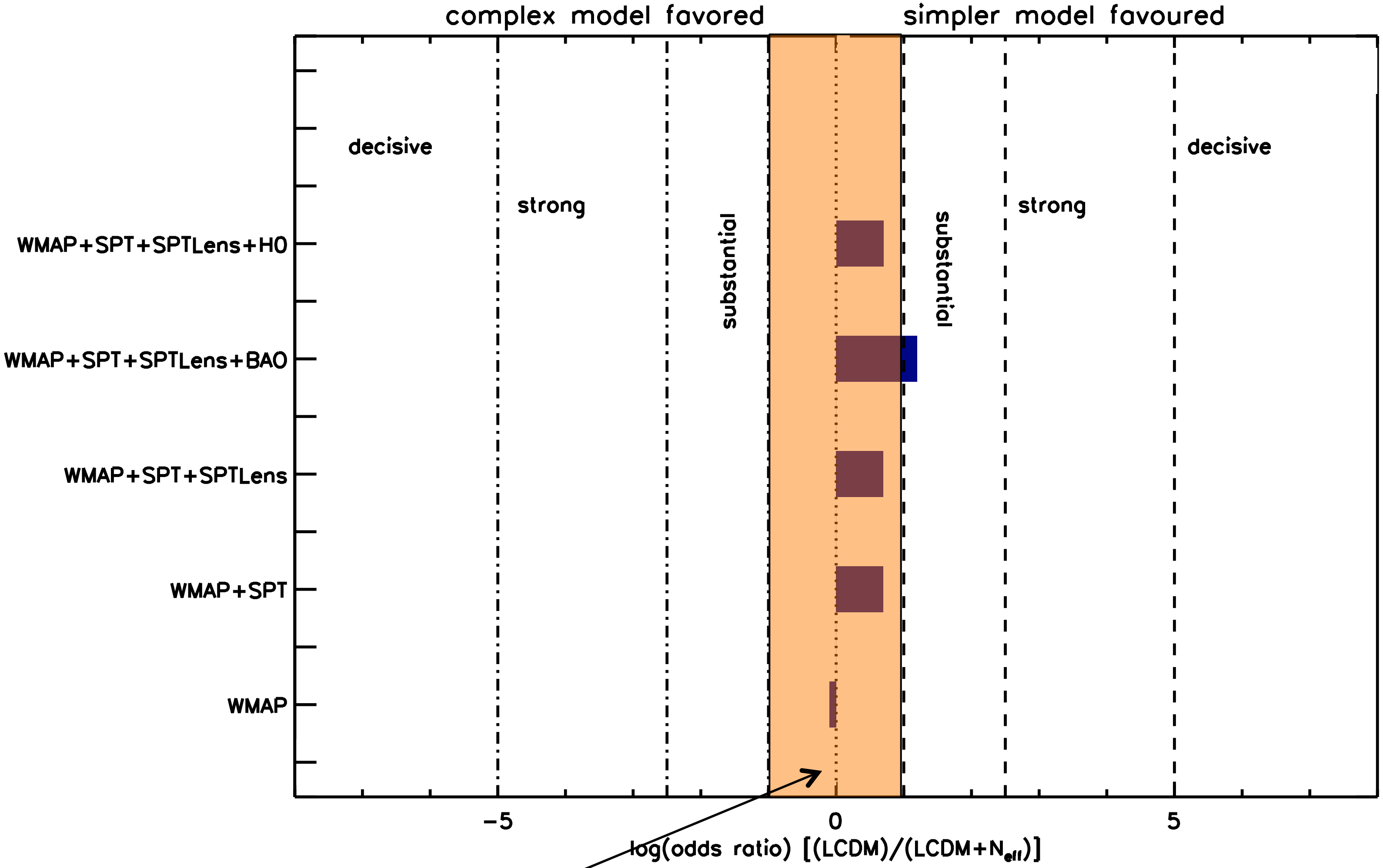
$$P(H \mid D) = \frac{P(D \mid H)P(H)}{P(D)}$$ Bayes

$$P(\boldsymbol{\theta} \mid D, H) = \frac{P(D \mid \boldsymbol{\theta}, H)P(\boldsymbol{\theta}|H)}{P(D \mid H)}$$ Bayes for parameter estimation

$$\mathcal{E} \equiv P(D \mid H) = \int d^n\,\boldsymbol{\theta} P(D \mid \boldsymbol{\theta}, H)P(\boldsymbol{\theta} \mid H)$$ Bayes for model itself

Computationally expensive! (there are packages to help out there e.g. cosmonest)

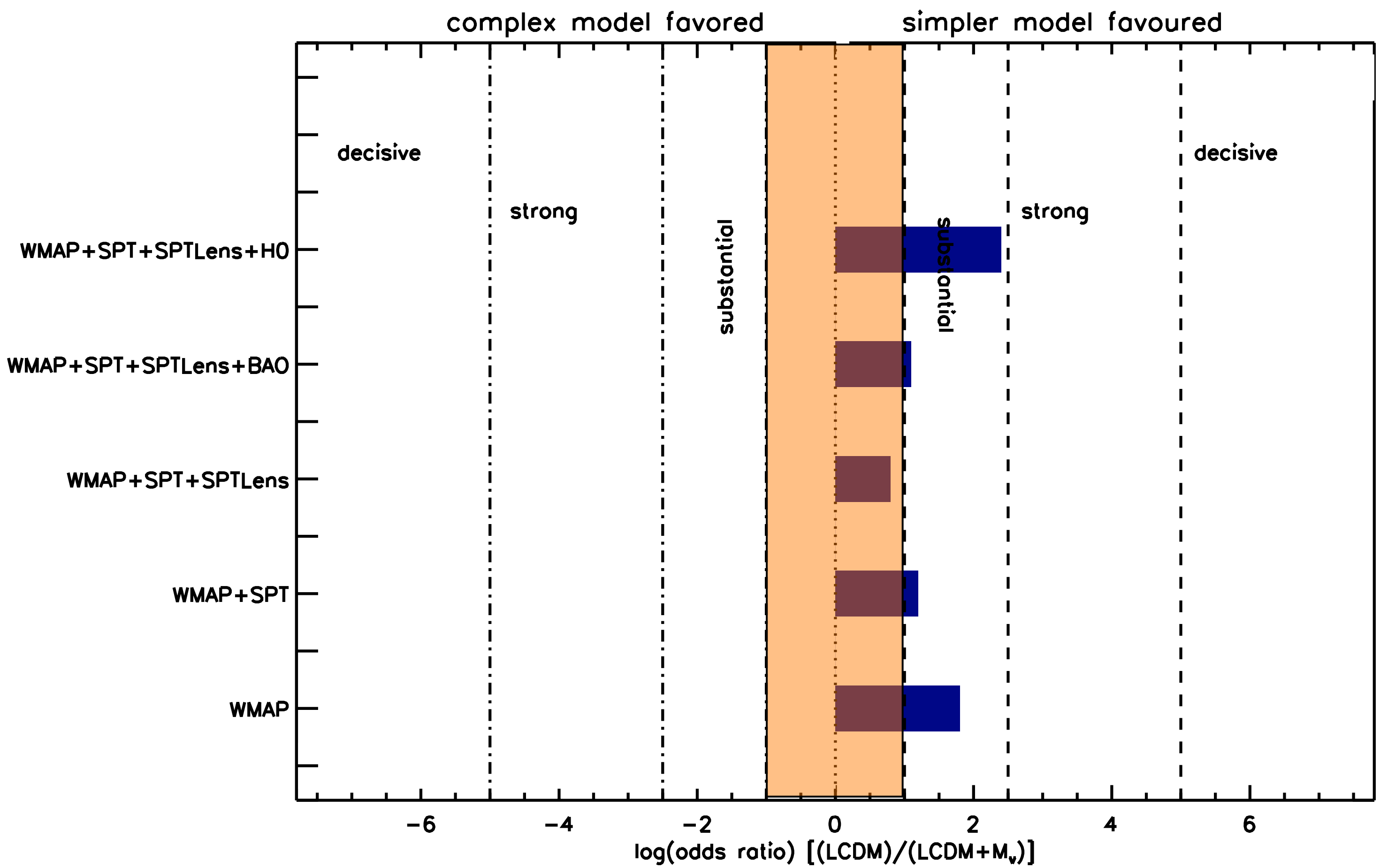
Evidence



undecisive

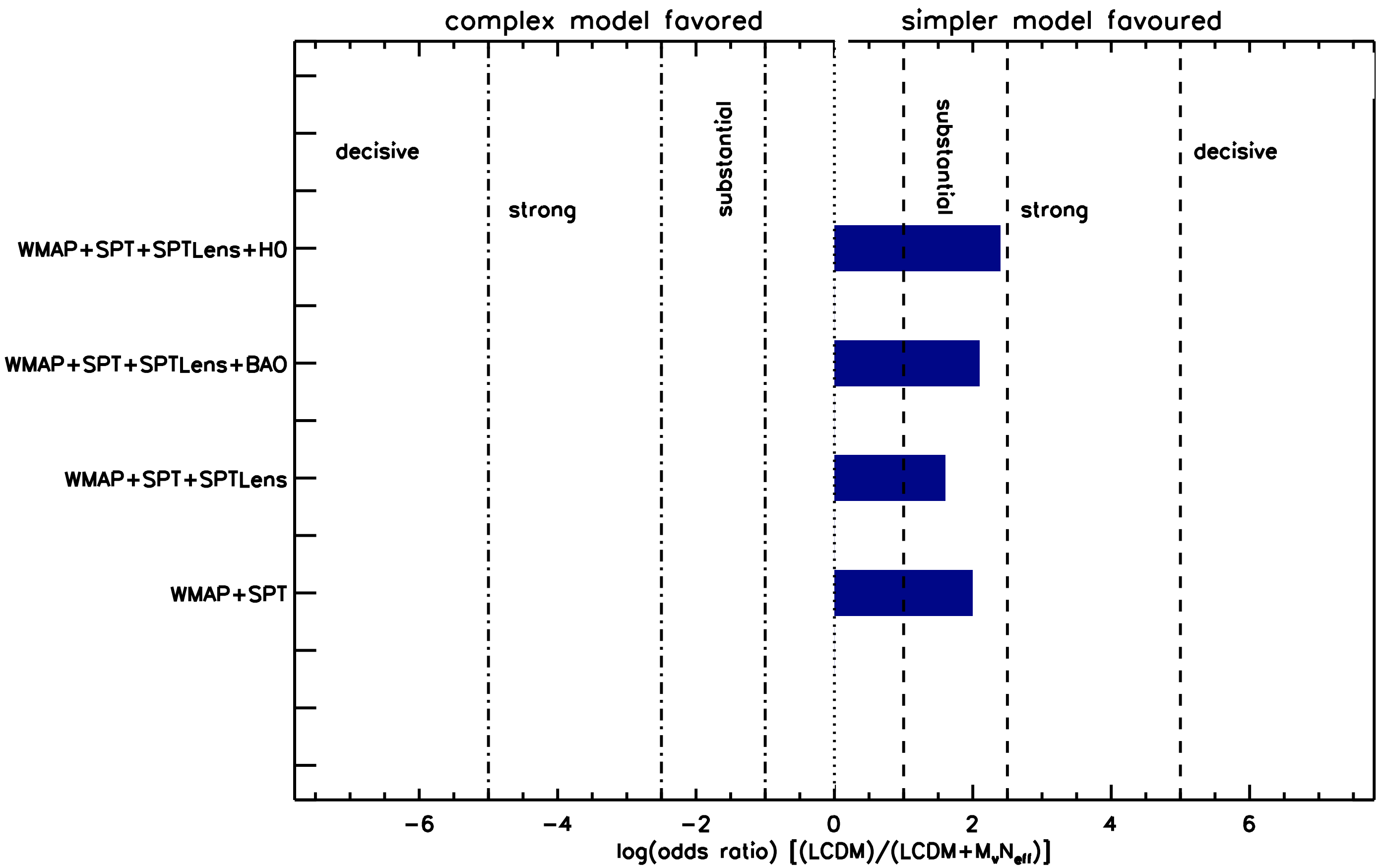
Feeney, Peiris, Verde in prep.

Evidence
(just for curiosity, Mν)



Feeney, Peiris, Verde in prep.

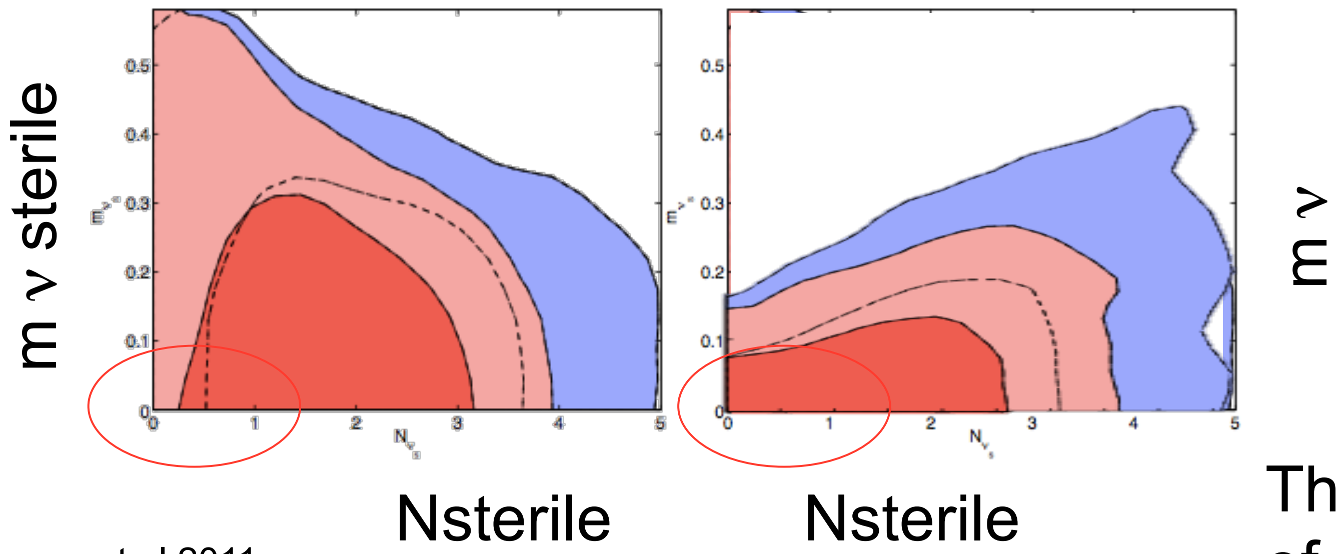
Evidence
(just for curiosity, Neff+ Mν)



Feeney, Peiris, Verde in prep.

In summary:

- Neff consistent with 3 (but also with 4) at 2 σ
- These are “light” neutrinos (<0.5 eV)
- more wiggle room: go beyond the minimal LCDM (errors gets slightly larger, but… epicycles)
- Avoid thermalization (some v. radical options)



Giusarma et al 2011

The pros and cons of being Bayesian

More forecasts

Carbone LV et al (2012)

Two surveys strategies

- A EUCLID like survey (slitless spectroscopy of $H\alpha$ emission lines)
- H-band magnitude limited survey , multislit spectroscopy (WFIRST)

Table 5: $\sigma(M_\nu)$ and $\sigma(N_{\rm eff})$ marginalised errors from LSS+CMB

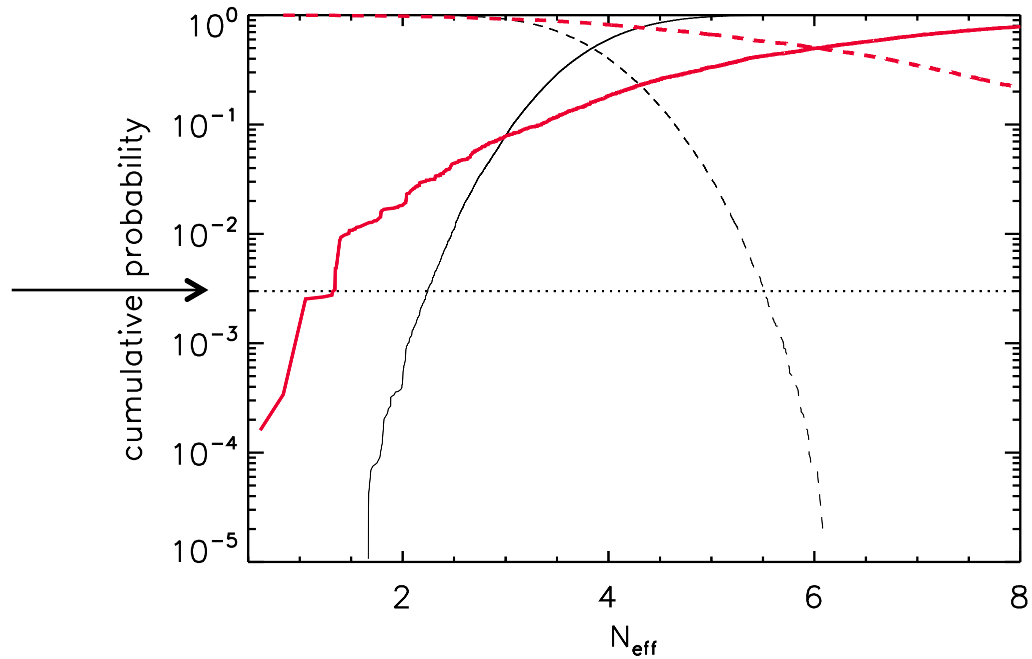
General cosmology						
fiducial→	M_ν=0.3 eV[a]	M_ν=0.2 eV[a]	M_ν=0.125 eV[b]	M_ν=0.125 eV[c]	M_ν=0.05 eV[b]	$N_{\rm eff}$=3.04[d]
slitless+BOSS+Planck	0.035	0.043	0.031	0.044	0.053	0.086
multi-slit+Planck	0.030	0.038	0.027	0.039	0.046	0.082
ΛCDM cosmology						
slitless+BOSS+Planck	0.017	0.019	0.017	0.021	0.021	0.023
multi-slit+Planck	0.015	0.016	0.014	0.018	0.018	0.019

[a]for degenerate spectrum: $m_1 \approx m_2 \approx m_3$; [b]for normal hierarchy: $m_3 \neq 0$, $m_1 \approx m_2 \approx 0$
[c]for inverted hierarchy: $m_1 \approx m_2$, $m_3 \approx 0$; [d]fiducial cosmology with massless neutrinos

On the other hand…

the cosmic neutrino background has been detected at >> 4 σ

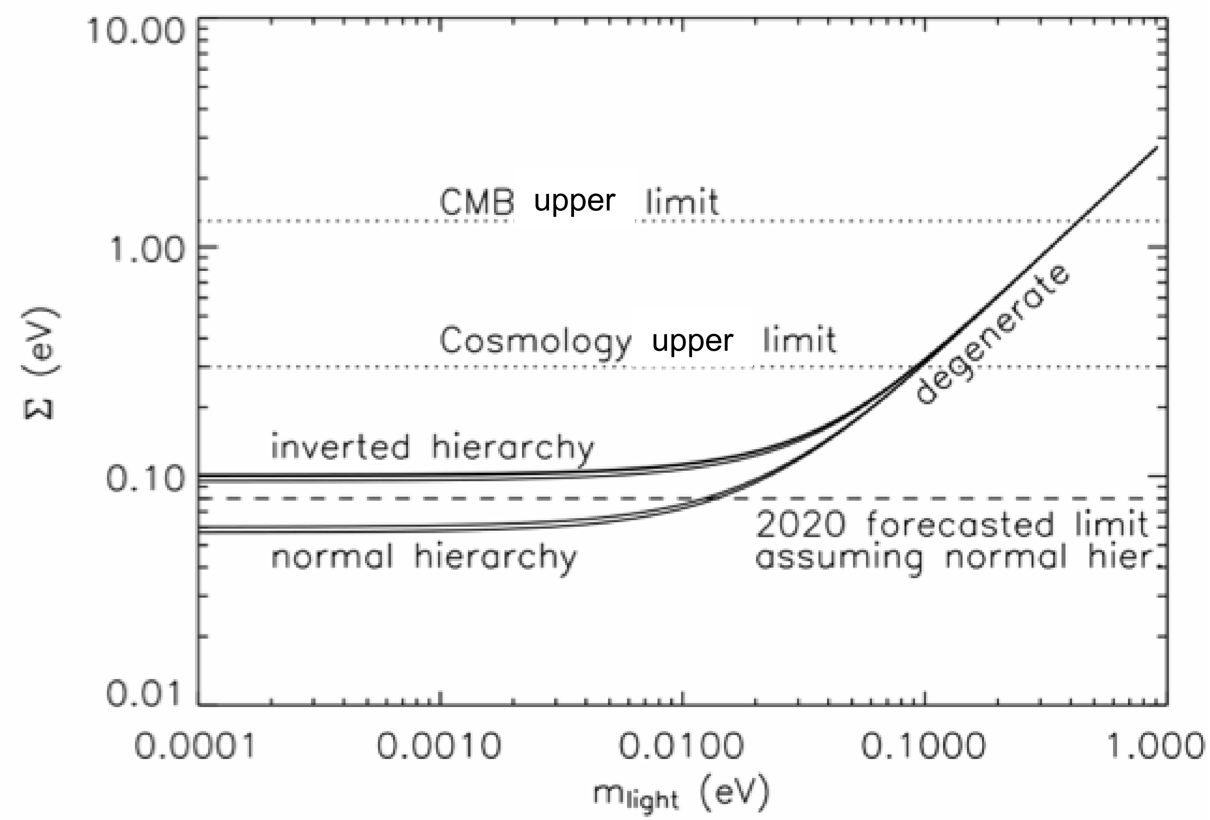
99.7%
or 0.003



WMAP

SPT

Outlook towards the future

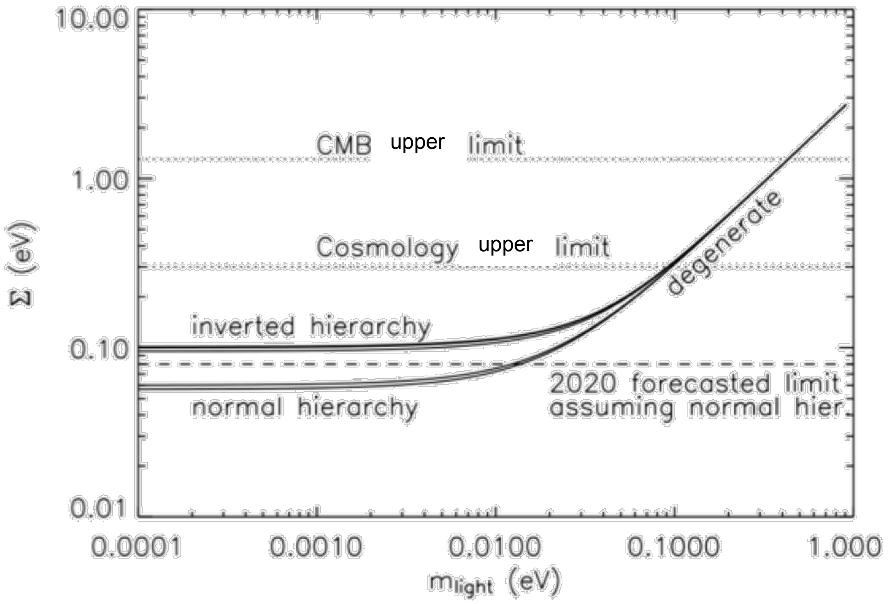


Can the hierarchy be determined?

Are neutrino Majorana or Dirac?

Jimenez, Kitching, Penya-Garay, Verde, 2010

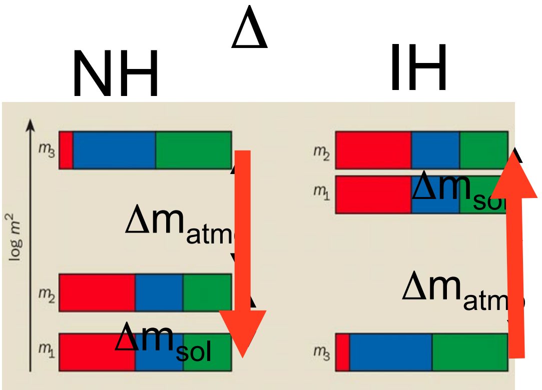
Can the hierarchy be determined?



Is there enough information in the sky? (ultimate experiment)

Can this be done with a specific survey?

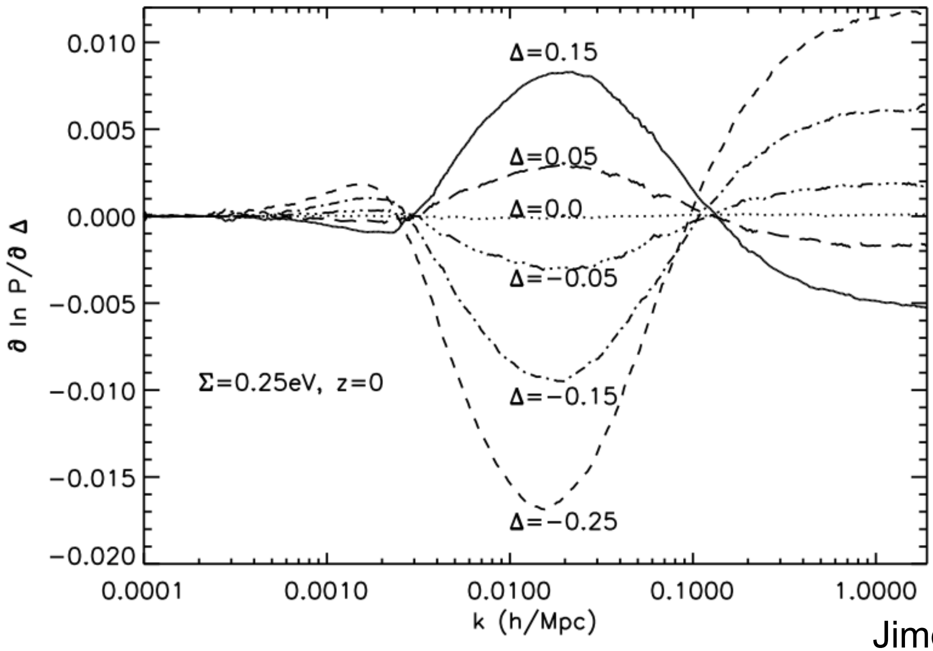
Hierarchy effect on the shape of the linear matter power spectrum

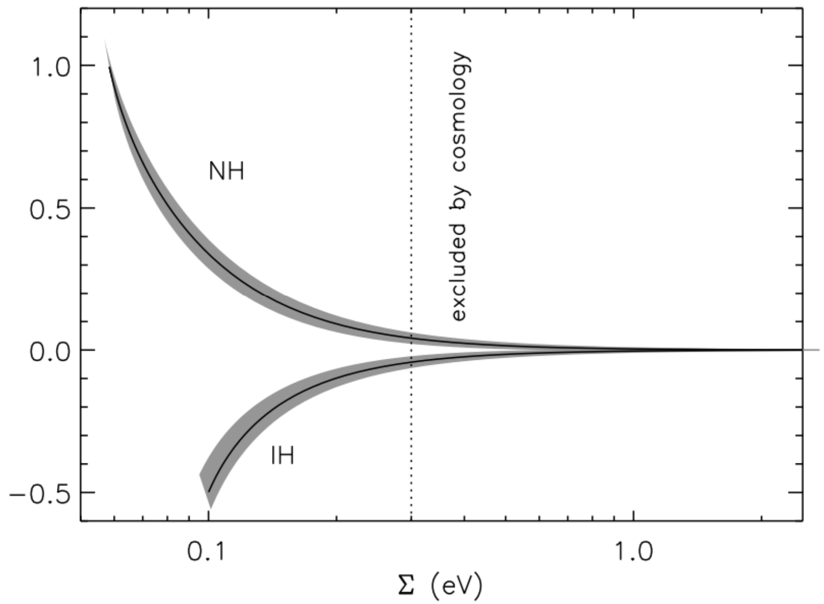


Neutrinos of different masses have different transition redshifts from relativistic to non-relativistic behavior, and their individual masses and their mass splitting change the details of the radiation-domination to matter- domination regime.

$$\text{NH}: \quad \Sigma = 2m + M \quad \Delta = (M - m)/\Sigma$$

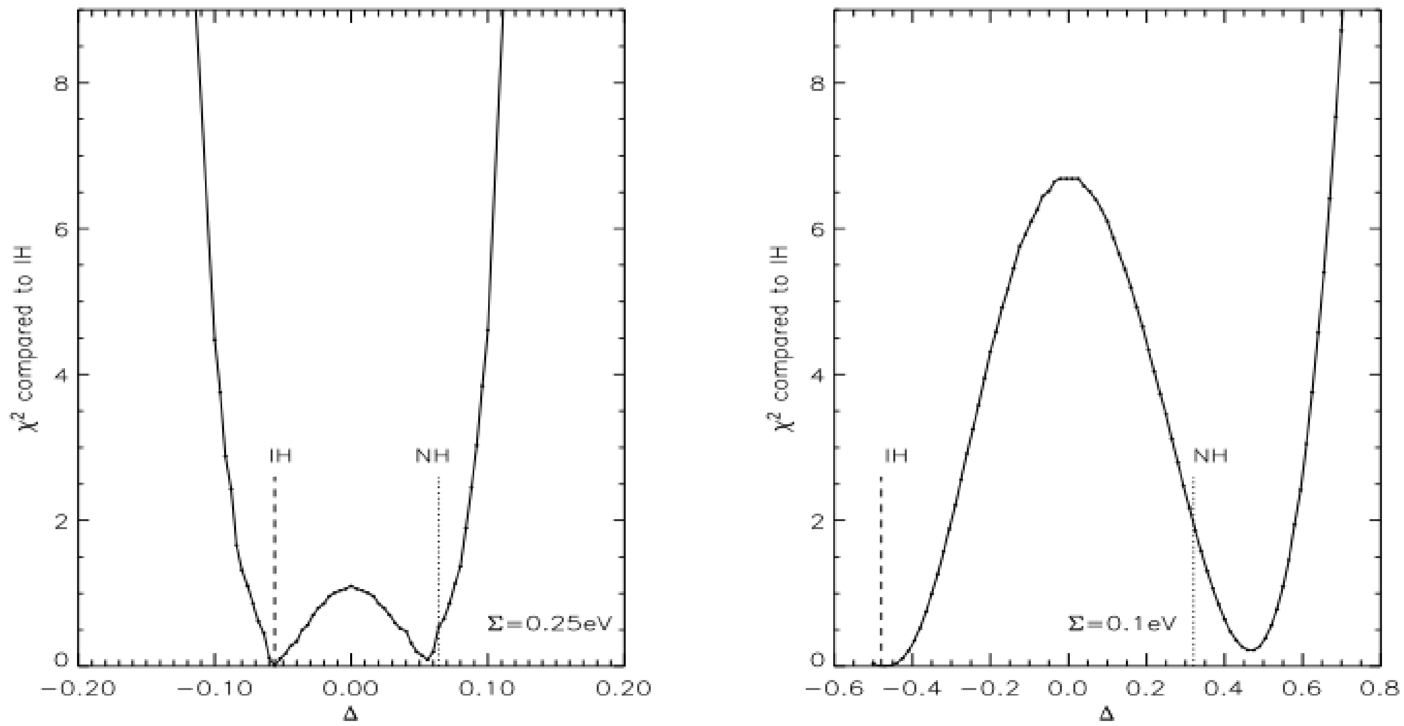
$$\text{IH}: \quad \Sigma = m + 2M \quad \Delta = (m - M)/\Sigma$$





Jimenez, Kitching, Penya-Garay, Verde, JACP2010

Hierarchy effect on the overall likelihood function



Jimenez, Kitching, Penya-Garay, Verde, JCAP 2010

A word of warning!

Lessons so far:

Should be careful about parameterization choice

Use information from oscillations

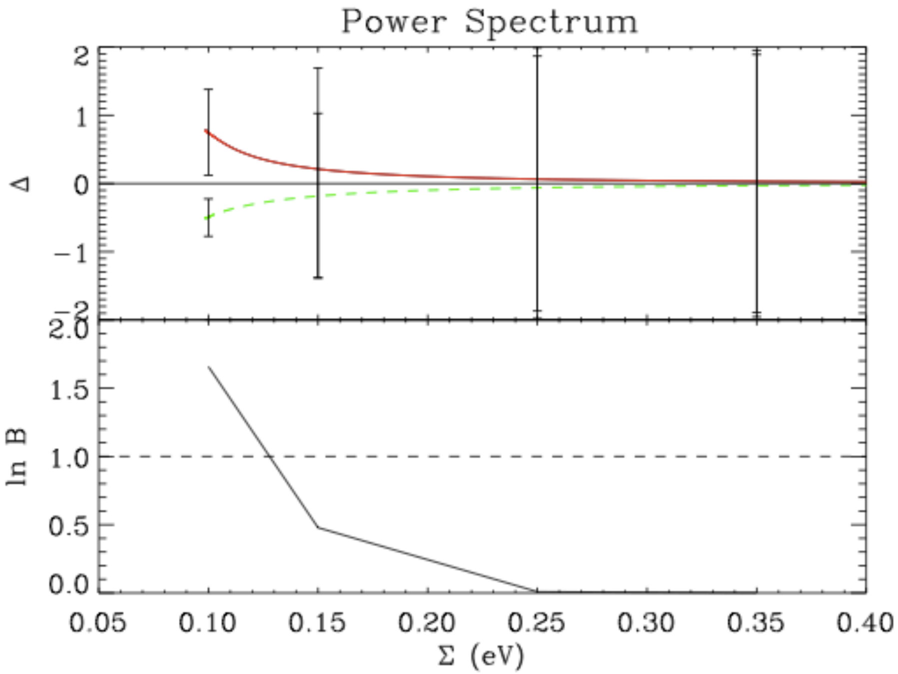
Cosmology is (mostly) sensitive to $|\Delta|$

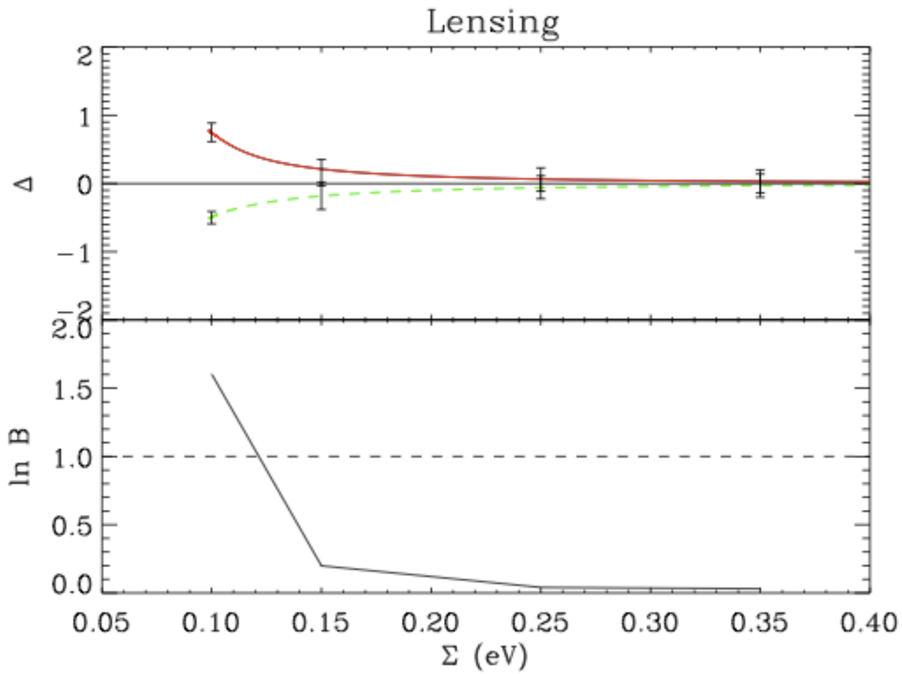
Fisher estimates are great but have limitations!

Natural question to ask:

What would it take to measure Δ?

Basically, the ultimate experiment



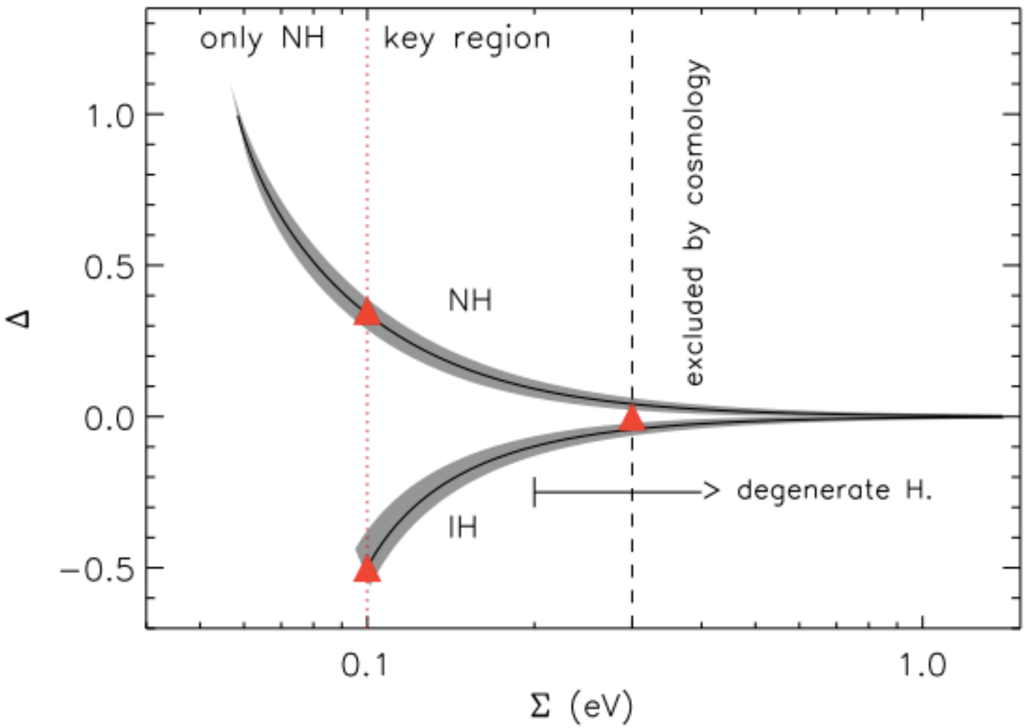


600 Gpc^3 at $z = 2$ and 2000 Gpc^3 at $z = 5$.

40,000 sq. deg. median redshift of 3.0
number density of 150 galaxies per □'

B is the Bayesian evidence ratio

What about non-linearities? simulations



Gadget 2

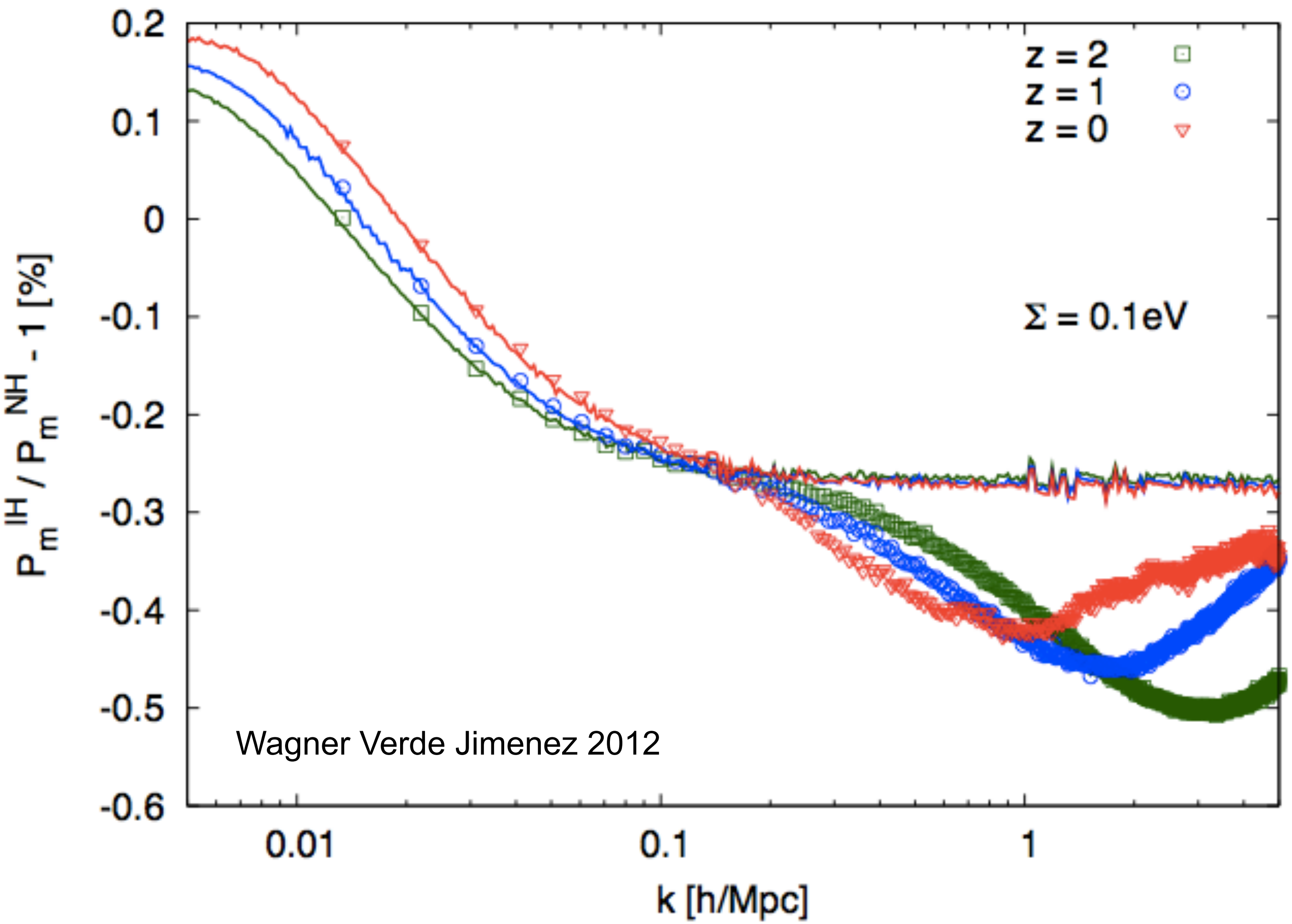
Small test runs to optimize time steps, varying initial z and number of tracers

Wagner LV Jimenez 2012

V=(600h/Mpc)^3 1billion CDM particles and 2 billion ν's

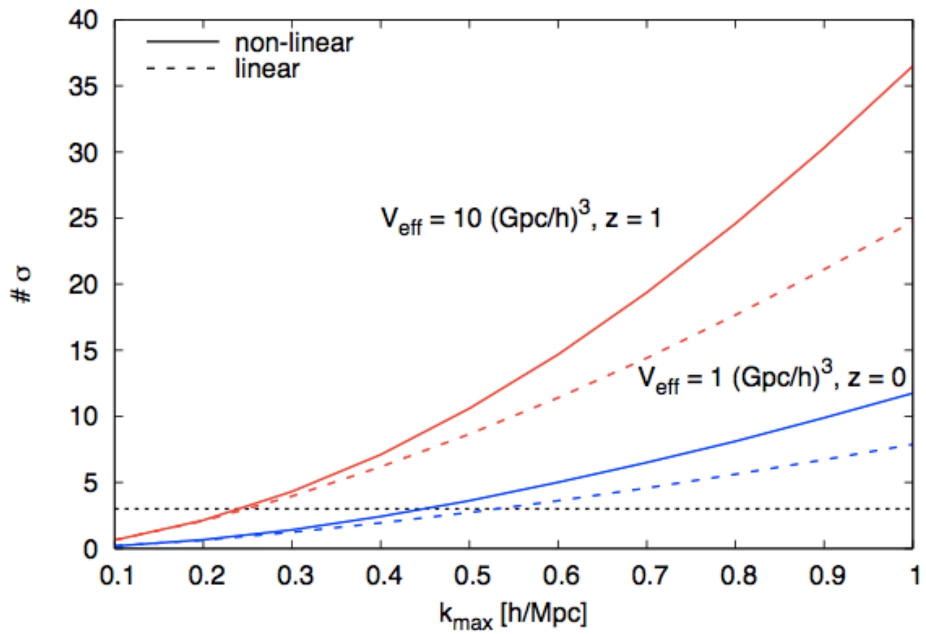
LDCM cosmology

Effect of Δ



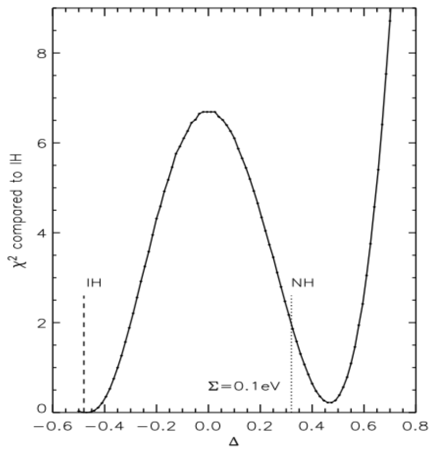
Non-linearities enhance the effect!

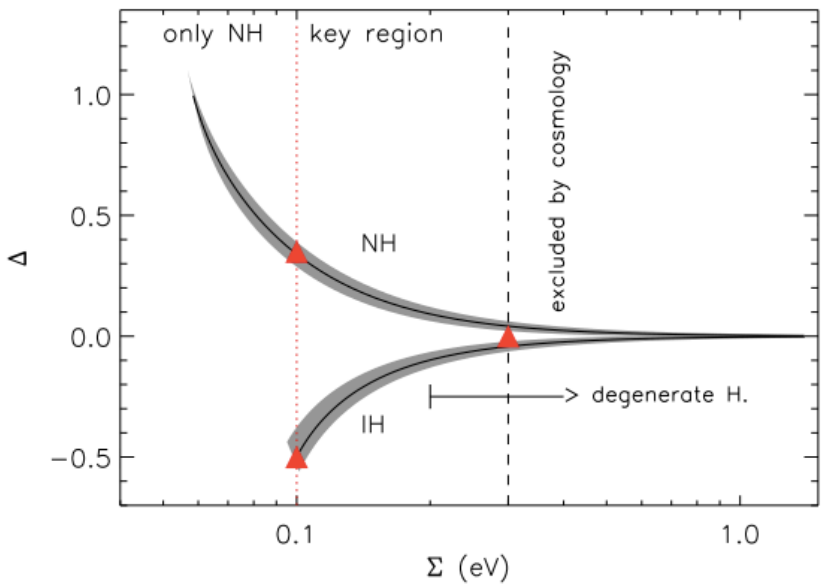
Detectability



NH vs IH

Keeping all other parameters fixed!





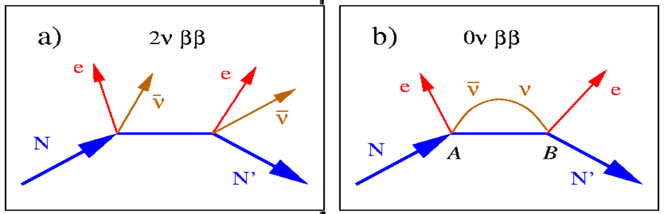
There's more parameter space!

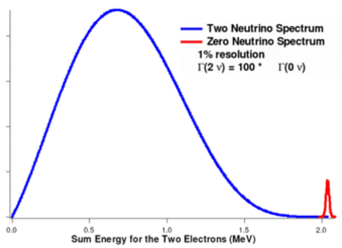
Dirac or Majorana? ⟺ hierarchy

Are neutrinos their own anti-particle?(are they Majorana or Dirac?)

$0\nu\beta\beta$ (next generation)

Yes

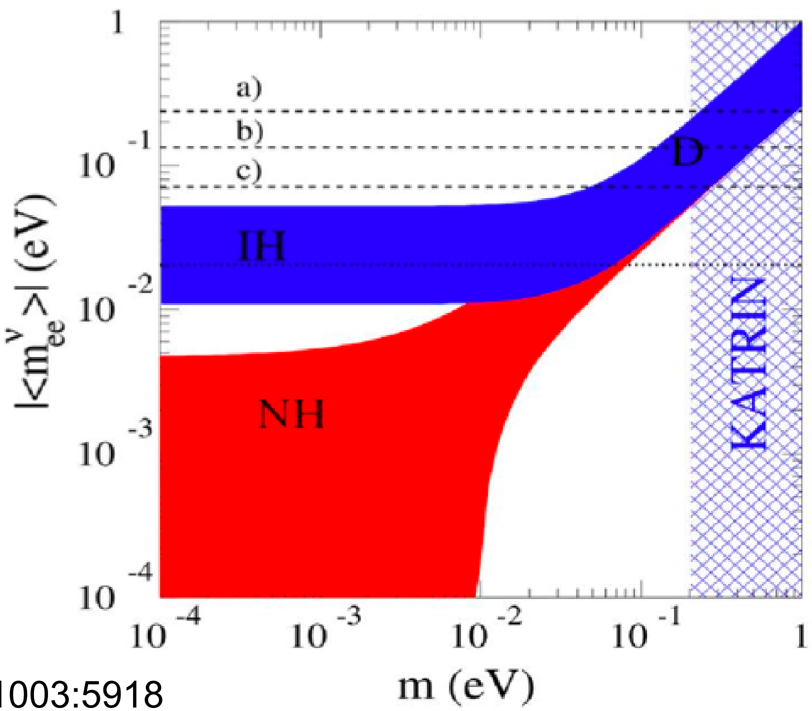




Majorana

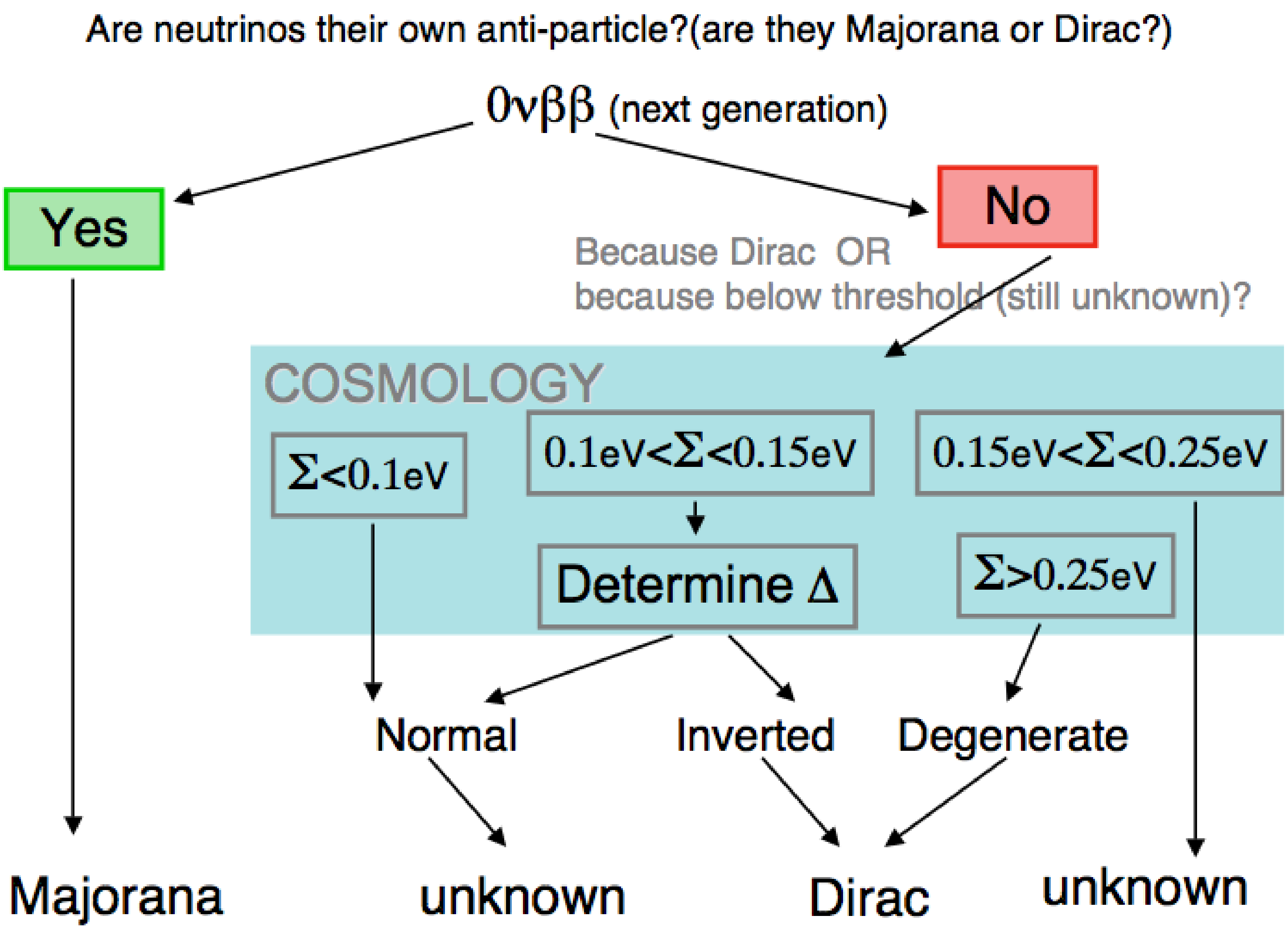
No

Because Dirac OR
because below threshold (still unknown)?



Jimenez, Kitching, Penya-Garay, Verde, arXiv:1003:5918

Future surveys can help!



Jimenez, Kitching, Penya-Garay, Verde, arXiv:1003:5918

Food for thoughts

This is a sub % effect!

Different linear Einstein-Boltzmann codes (e.g. CAMB (Lewis et al. 2000) and CLASS (Blas et al. 2011)) still do not agree to 0.1% precision on the relevant scales. (this is a much easier problem!)

We have quantified the effect on the RELATIVE quantity: much more robust against numerical errors.
Even without massive neutrinos, it is challenging to compute the non-linear power spectrum to sub-percent precision.

At small scales baryonic effects dominate!

Effect of degeneracy with cosmology (de Putter et al, in preparation)

Will soon be possible to constrain $|\Delta|$, powerful consistency check!!!

Conclusions

- Precision cosmology means that we can start (or prepare for) constraining interesting physical quantities.
- Neutrino properties: absolute mass scale, number of families, maybe hierarchy
- $\Sigma m\nu < 0.3$ eV (95%)
- N_{eff} consistent with 3.04 (error of ~1 , 95%)
- The cosmic neutrino background is there!
- Large future surveys means that sub % effects become detectable, which brings in a whole new set of challenges and opportunities (e.g., mass hierarchy)

END

Robust neutrino constraints…

Beth Reid, LV, R. Jimenez, Olga Mena,(JCAP 2010)

DATA: WMAP5 H0 from Riess et al 2009 h=0.74+-0.036

MaxBCG $\sigma_8(\Omega_m/0.25)^{0.41} = 0.832 \pm 0.033.$ Rozo et al 09, Koester et al 07, Johnston et al 07

SDSS DR7 halo P(k)

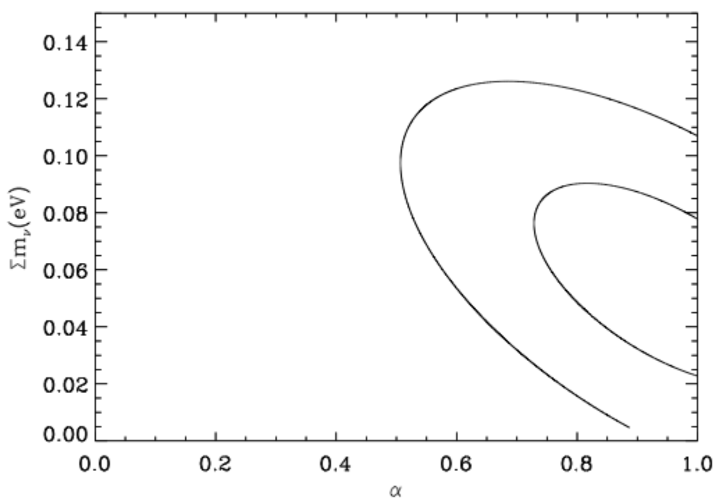
Bayesian and Frequentist upper 95% C.L. bound on $\sum m_\nu$

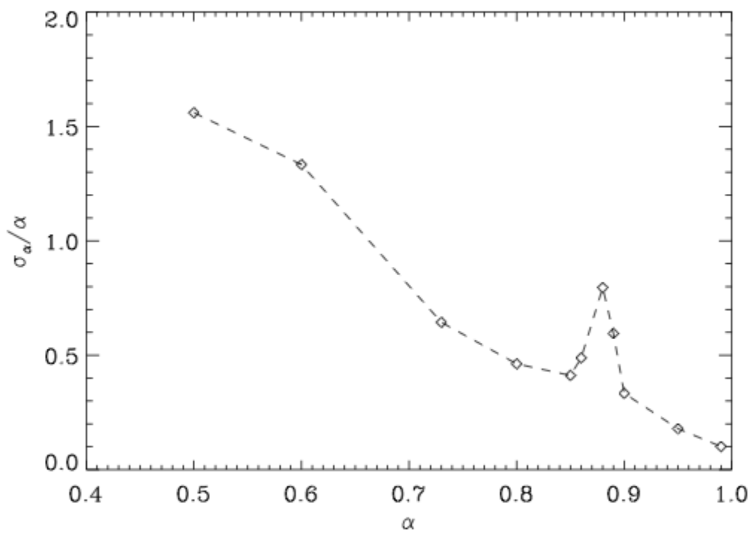
model	base dataset	–	+maxBCG	$+H_0$	$+$maxBCG$+H_0$	
ΛCDM	WMAP5	1.3	1.1	0.59	0.40	Bayesian
ΛCDM	WMAP5+BAO+SN	0.67	0.35	0.59	0.31	
ΛCDM $+\alpha$	WMAP5	1.34	1.25	0.54	0.39	
ΛCDM $+r$	WMAP5	1.36	1.18	0.83	0.40	
wCDM	WMAP5+BAO+SN	0.80	0.52	0.72	0.47	
dark coupling	WMAP5+$\hat{P}_{halo}(k)$+SN	-	-	0.51	-	
ΛCDM	WMAP5	1.3	1.0	0.57	0.41	Frequentist
ΛCDM	WMAP5+BAO+SN	0.71	0.41	0.61	0.30	
ΛCDM $+\alpha$	WMAP5	1.28	1.17	0.63	0.43	
ΛCDM $+r$	WMAP5	1.23	0.86	0.72	0.30	
wCDM	WMAP5+BAO+SN	0.82	0.46	0.74	0.44	
dark coupling	WMAP5+$\hat{P}_{halo}(k)$+SN	-	-	0.56	-	

aside

$$m_3 = \alpha \sum m_\nu . \qquad m_1 = m_2$$

De Bernardis et al. 09,
Slosar 06



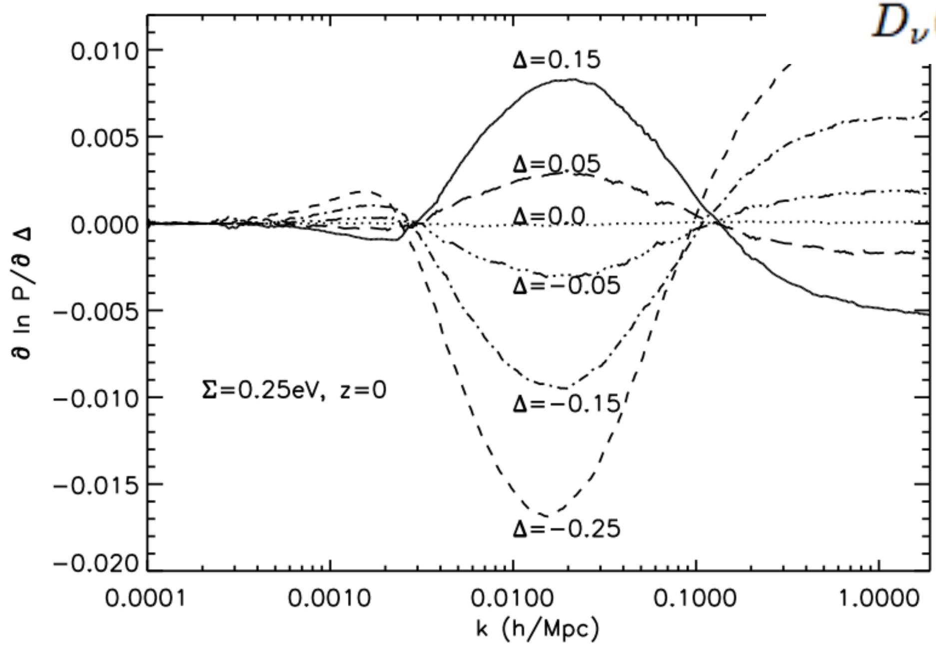


In details….

$$\frac{k^3 P(k;z)}{2\pi^2} = \Delta_R^2 \frac{2k^2}{5H_0^2\Omega_m^2} D_\nu^2(k,z) T^2(k) \left(\frac{k}{k_0}\right)^{(n_s-1)}$$

$$k_{\mathrm{fs},i} = 0.113 \left(\frac{m_{\nu_i}}{1\mathrm{eV}}\right)^{1/2} \left(\frac{\Omega_m h^2}{0.14} \frac{5}{1+z}\right)^{1/2} \mathrm{Mpc}^{-1}$$

$$\begin{aligned} D_\nu(k,z) &= D(k,z) && k < k_{\mathrm{fs},m} \\ D_\nu(k,z) &= (1-f_{\nu,m}) D(z)^{(1-p_m)} && k_{\mathrm{fs},m} < k < k_{\mathrm{fs},\Sigma} \\ D_\nu(k,z) &= (1-f_{\nu,\Sigma}) D(z)^{(1-p_\Sigma)} && k > k_{\mathrm{fs},\Sigma}\,, \end{aligned}$$



$$\begin{aligned} \mathrm{NH}: &\quad \Sigma = 2m + M \quad \Delta = (M-m)/\Sigma \\ \mathrm{IH}: &\quad \Sigma = m + 2M \quad \Delta = (m-M)/\Sigma \end{aligned}$$